Suggested Strategies for Infinite Series ($\sum_n a_n$)

(which work SUM of the time)

Note 1: there are only two series we've discussed you can sum analytically

a) Geometric Series with $|r|<1$; $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$

b) Partial fraction series which "telescope" in to parial sums whose $n \to \infty$ limit you can evaluate

Note2: All of the <u>comparison</u> tests { dominated convergence, limit comparison, integral test, ratio/root tests } are for |absolute| convergence,

Note3: Liebniz's Theorem establishes the <u>convergence</u> of an <u>alternating</u> series it does <u>Not</u> establish whether the convergence is <u>absolute</u> or <u>conditional</u>.

Note4: if $\lim_{n\to\infty} \frac{|a_n|}{b_n} = 0$, then $\sum_n^{\infty} b_n$ <u>must</u> converge to establish $\sum^{\infty} |a_n|$ converges

$\lim_{n\to\infty} \frac{|a_n|}{b_n} = \infty$, then $\sum_n^{\infty} b_n$ <u>must</u> diverge to establish $\sum^{\infty} |a_n|$ diverges

if $\lim_{n\to\infty} \frac{|a_n|}{b_n} = 0$ and $\sum^{\infty} b_n$ diverge <u>or</u> $\lim_{n\to\infty} \frac{|a_n|}{b_n} = \infty$ and $\sum^{\infty} b_n$ converge

Nothing is established (it's like ratio/root test of 1)