Exam 3a Calculus II 804-232-001 Fall 2009 /100

Name ______________________

All problems are worth 20 points.

1. Evaluate the following sums : .

a) $\sum_{n=0}^{\infty} \frac{1}{n^2+11n+30}$ $=$ ______________________

b) $\sum_{j=0}^{\infty} \frac{(\frac{3\pi}{4})^{2j+1}(-1)^j}{(2j+1)!}$ $=$ ______________________

2. For each of the following indicate (with a valid justification) whether the series is absolutely convergent, conditionally convergent, or divergent.

a) $\sum_{n=2}^{\infty} (-1)^n \left(1 - \sqrt{1 - \frac{2}{n^2}}\right)$

b) $\sum_{n=0}^{\infty} \frac{(-10)^n (n)!}{(2n)!}$

3. For each power series below : (1) Determine the interval of absolute convergence.
(2) If the radius of convergence is finite, determine if the series converges at the endpoints.

$\sum_{n=0}^{\infty} \frac{(-x)^n}{\sqrt{n^2+8}}$

4. Express each function below as a power series in x . State enough terms so that the pattern is apparent.

$a)$ $x^4 e^{-x^2} =$

b) $\int_0^x t^4 e^{-t^2} dt =$

5. For the problem below evaluate the limit by the use of an appropriate series.

$$\lim_{x \to 0} \frac{1 - 2x^2 - \cos(2x)}{x^4}$$

10 point Bonus.

Consider the sequence defined by the following recurrence relation :

$$a_0 = 1 \;; \qquad a_{n+1} = -\frac{x}{n+1} a_n$$

a) Evaluate the following : $a_1 =$ ______________________ $a_2 =$ ______________________

$a_3 =$ ______________________ $a_{10} =$ ______________________

b) For what values of x does $\sum_{n=0}^{\infty} a_n$ converge absolutely?

c) Evaluate the series at $x = 2$.