Calculus II 804-232-001 Final Exam Fall 2006 /200

Name ____________________

Each problem is worth 12.5 points.
Evaluate the following integrals. To get full credit you must show some work. If you use a standard formula, please state it in general form before plugging in values If you use a formula from a table, please indicate this.

1. $\int_0^{\infty} xe^{-x}\,dx$ $=$ ____________________

2. $\int \frac{dx}{\sqrt{9x^2-18x+25}}$ $=$ ____________________

3. $\int \frac{x^3-2x^2+1}{x^3-x}\,dx$ $=$ ____________________

4. Solve the following ODE subject to the stated initial condition.

$\frac{dU}{dx} + 2U = 1$ with $E(0) = 3$.

5. Set up (but **don't evaluate**) the definite integral which gives the total arclength of the curve $y = e^x$ from $x = 0$ to $x = 2$.

6. Evaluate the following sum :

$$\sum_{n=1}^{\infty} \frac{1}{n}\left(-\frac{1}{4}\right)^n \qquad = \underline{\hspace{6cm}}$$

Indicate (with a valid justification) whether the series is absolutely convergent, conditionally convergent, or divergent.

7. $$\sum_{j=1}^{\infty} \frac{\sin(\frac{j\pi}{2})}{2j-1}$$

Determine the interval of absolute convergence. If the radius of convergence is finite, determine convergence at the endpoints.

8. $$\sum_{n=1}^{\infty} \frac{(-4x)^n (2n-1)!}{(2n+1)!}$$

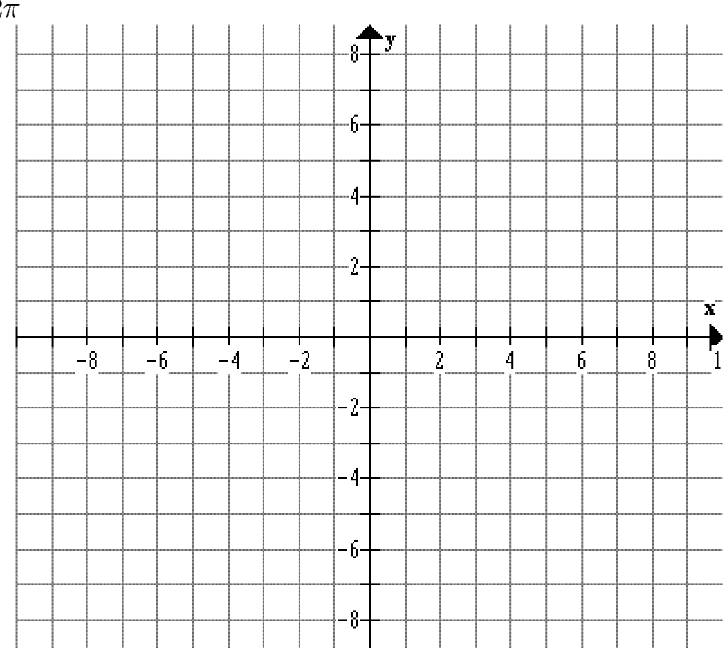
9. Express the function below as a power series in x . State enough terms so that the pattern is apparent.

$$\frac{d}{dx}\left(e^{-x^2}\right) \quad = \underline{\hspace{10cm}}$$

For the following curve :

(1) Identify which conic section or degenerate case it represents.
(2) Calculate (if not a parabola) *a* , *b* and *c* .
(3) Calculate the eccentricity, e .
(4) Calculate the coordinates of all vertices and/or centers.
(5) Calculate the coordinates of all foci.
(6) Sketch the curve, showing asymptotes if it is a hyperbola.

10. $x = 2\sin(t) - 2$
$y = 3\cos(t) + 3$ for $0 \le t < 2\pi$



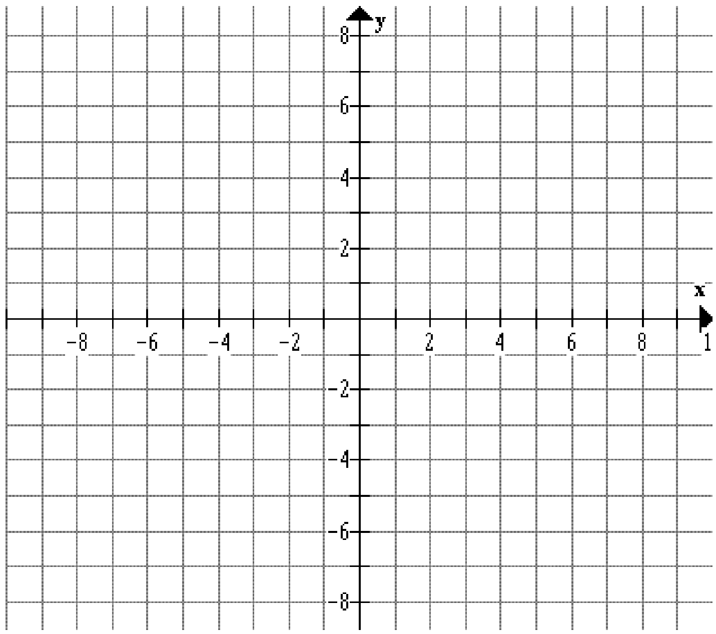
11. For the curve of problem 10 determine the equation of the tangent line at the point $\left(-2-\sqrt{2}, 3+\frac{3}{2}\sqrt{2}\right)$.

12. Write the polar equation of a conic section having the given directrix, focus and eccentricity.

Focus : $(0\ ,0)$ Directrix : $y = 3$ Eccentricity $= \frac{3}{2}$

13. Find the equation of the plane containing the points $(2, -1, 3)$, $(-1, 1, -3)$, and $(4, 4, -1)$.

14. Sketch the following polar curve $r = 4\cos(3\theta)$



15. Set up **and evaluate** a definite integral which gives the area enclosed by the "petal" alligned along the positive x axis in the figure of problem 14.

16. Given $\vec{\boldsymbol{A}} = 2\widehat{\boldsymbol{i}} + 3\widehat{\boldsymbol{j}} - 2\widehat{\boldsymbol{k}}$, and $\vec{\boldsymbol{B}} = 2\widehat{\boldsymbol{i}} - 2\widehat{\boldsymbol{j}} - 3\widehat{\boldsymbol{k}}$ evaluate the following :

a) $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}$ = ______________________________

b) $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$ = ______________________________