

Name _____

1. Given the following functions, compute the derivatives. (32 points)

a) $f(x) = \frac{1}{\sqrt{1-x^2}}$ $f'(x) =$ _____

b) $f(x) = \frac{\tan(\alpha x)}{x}$ $f'(x) =$ _____

c) $f(x) = \ln(8x^5e^{-3x})$ $f'(x) =$ _____

d) $f(x) = \tan^{-1}(e^{-x})$ $f'(x) =$ _____

e) $f(x) = \sqrt[3]{\frac{x^6e^{-9x}}{x^2+1}}$ $f'(x) =$ _____

f) $f(x) = \cos^{-1}(1-x^2)$ $f'(x) =$ _____

g) $f(x) = \int_{-x}^x \frac{\cos(2\theta)}{\sqrt{4+\theta^2}} d\theta$

$f'(x) = \underline{\hspace{10em}}$

h) $f(x) = \int_0^{\sin(x)} \sin^{-1}(t) dt$

$f'(x) = \underline{\hspace{10em}}$

2. Evaluate the following integrals. (40 points)

a) $\int \frac{\tan(x)}{\cos^2 x} dx$

$= \underline{\hspace{10em}}$

b) $\int \frac{1}{\sqrt{25-9z^2}} dz$

for $|z| < \frac{5}{3}$

$= \underline{\hspace{10em}}$

c) $\int \frac{z}{\sqrt{25-9z^2}} dz$

for $|z| < \frac{5}{3}$

$= \underline{\hspace{10em}}$

d) $\int \frac{1}{25-9z} dz$

$= \underline{\hspace{10em}}$

e) $\int_0^3 \frac{1}{25+9z^2} dz$ = _____

f) $\int_0^{\ln 2} \frac{e^x}{9+e^x} dx$ = _____

g) $\int \frac{9x^8-6x^6+4x^4-2x^2}{x^4} dx$ = _____

h) $\int_0^{\frac{\pi}{2}} e^{-\cos(\theta)} \sin(\theta) d\theta$ = _____

i) $\int_0^{\frac{\pi}{2}} \frac{\sin(x)\cos(x)}{1+\sin^2(x)} dx$ = _____

j) $\lim_{a \rightarrow \infty} \int_0^a \frac{1}{4+a^2x^2} dx$ = _____

3. Evaluate the following. (28 points).

a) $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x^2} = \underline{\hspace{2cm}}$

b) $\lim_{x \rightarrow 0} \frac{e^{\beta x} - 1}{x} = \underline{\hspace{2cm}}$

c) $\lim_{x \rightarrow 0} \frac{\ln(1+x^2)}{\sqrt{x}} = \underline{\hspace{2cm}}$

d) $\lim_{x \rightarrow 0^+} x^x = \underline{\hspace{2cm}}$

e) $\lim_{x \rightarrow 0} \frac{\int_0^x e^{-t^2} dt}{x} = \underline{\hspace{2cm}}$

f) $\lim_{x \rightarrow \infty} \tanh(4x) = \underline{\hspace{2cm}}$

g) $\lim_{x \rightarrow \infty} \frac{\sqrt{e^{2x} + 4x + 10}}{e^x - 1000x} = \underline{\hspace{2cm}}$

4. (20 points)

a) Find the slope of the curve defined by $x^2 - y^2 + 2xy + 1 = 0$ at the point $(-2, 1)$.

b) Find the equation of the tangent line to the above curve at $(-2, 1)$.

c) Find $\frac{d^2y}{dx^2}$ at $(-2, 1)$.

5. (20 points)

The displacement s from the origin of a particle is given by the following function of time t :

(A negative displacement means a position to the left, a positive displacement is a position to the right.)

$$s(t) = 5 + 12t - 3t^2 \quad ; \quad \text{for } 0 \leq t \leq 3$$

a) Determine the velocity, v , as a function of time. $v =$ _____

b) Determine the acceleration, a , as a function of time. $a =$ _____

c) What is the net displacement, Δs , from $t = 0$ to $t = 3$? $\Delta s =$ _____

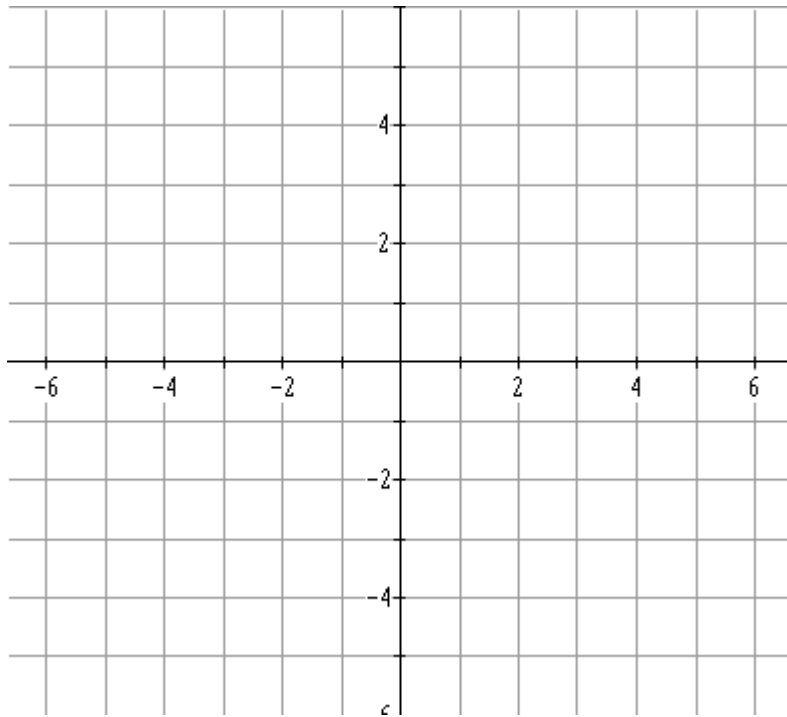
d) When is the particle moving to the left?

e) When is the particle not moving?

f) When is the particle moving the fastest?

6. Solve the following differential equations and sketch the solution. (20 points)

a) $\frac{df}{dx} = -f(x)$ with $f(0) = 4$



7. (20 points)

Given $y = f(x) = xe^{-x^2}$

a) Compute $f'(x)$ = _____

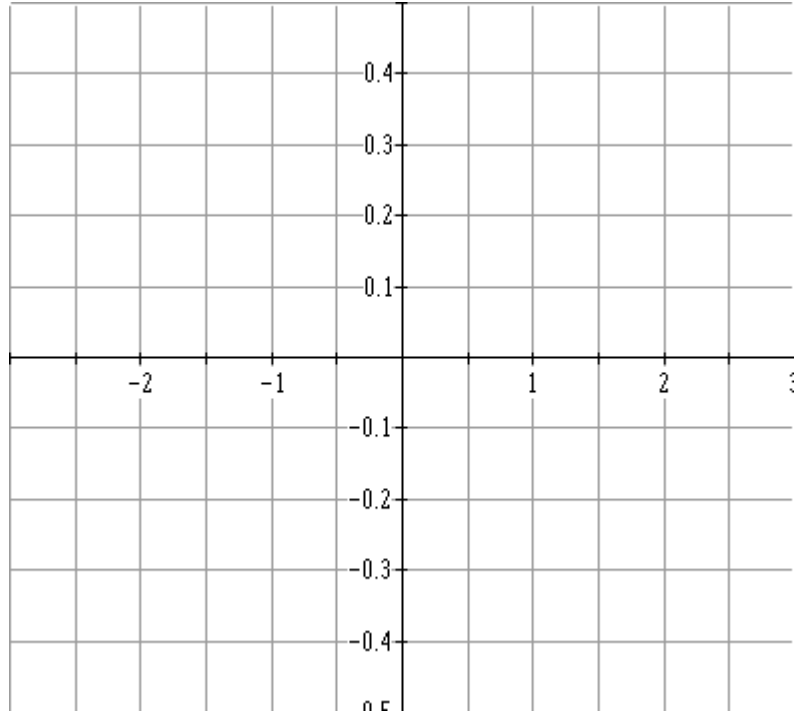
b) Compute $f''(x)$ = _____

c) Locate the roots of $f(x)$.

d) Find the location of all the maxima and minima of $f(x)$. Indicate which points are maxima (and whether they are absolute or only local) and are minima (absolute or local).

e) Find the location of all the inflection points of $f(x)$.

f) Sketch $f(x)$ on the next page.



8. (20 points)

a) Calculate the area of the region in the $x - y$ plane bounded by the curves $x = 2$, $y = x^3$, and $y = 0$.

b) Calculate the volume of the solid generated when the region in part a. is rotated about the x axis.

c) Calculate the volume of the solid generated when the region in part a. is rotated about the y axis.