Final Exam Calculus I 804-231-005 Spring 2009 /200

Name ________________

1. Given the following functions, compute the derivatives. (32 points)

a) $f(x) = \sqrt{1-2x^2}$ $f'(x)$ = ______________________

b) $f(x) = \frac{\tan(\alpha x)}{x}$ $f'(x)$ = ______________________

c) $f(x) = \ln\left(8x^5 e^{-3x}\right)$ $f'(x)$ = ______________________

d) $f(x) = \sin^{-1}(e^{-x})$ $f'(x)$ = ______________________

e) $f(x) = \sqrt[3]{\frac{x^6 e^{-9x}}{x^2+4}}$ $f'(x)$ = ______________________

f) $f(x) = \tan^{-1}(x^2)$ $f'(x)$ = ______________________

g) $f(x) = \int_{-x}^{x} \frac{e^y}{\sqrt{4+y^2}}\, dy$ $f'(x) =$ ______________________

h) $f(x) = \int_{0}^{\sin(2x)} \frac{\sin^{-1}(t)}{t^2+1}\, dt$ $f'(x) =$ ______________________

2. Evaluate the following integrals. (40 points)

a) $\int \frac{\tan(x)}{\cos^2 x}\, dx$ $=$ ______________________

b) $\int \frac{1}{\sqrt{25-9z^2}}\, dz$ for $|\,z\,| < \frac{5}{3}$ $=$ ______________________

c) $\int \frac{z}{\sqrt{25-9z^2}}\, dz$ for $|\,z\,| < \frac{5}{3}$ $=$ ______________________

d) $\int \frac{1}{25-9z}\, dz$ $=$ ______________________

e) $\int_0^3 \frac{1}{25+9z^2}\,dz$ = ______________________

f) $\int_0^{\ln 2} \frac{e^x}{7+4e^x}\,dx$ = ______________________

g) $\int \frac{9x^8-6x^6+4x^4-2x^2}{x^4}\,dx$ = ______________________

h) $\int_0^{\frac{\pi}{4}} e^{-\tan(\theta)}\ \frac{1}{\cos^2(\theta)}\,d\theta$ = ______________________

i) $\int_0^1 \frac{\sinh(x)\cosh(x)}{1+\sinh^2(x)}\,dx$ = ______________________

j) $\lim_{a\to\infty}\int_0^a e^{-x}\,dx$ = ______________________

3. Evaluate the following. (28 points).

a) $\lim_{x \to 0} \frac{1-\cos(2x)}{x^2}$ = ______________________

b) $\lim_{x \to 0} \frac{e^{\beta x}-1}{x}$ = ______________________

c) $\lim_{x \to 0} \frac{\ln(1+x^2)}{\sqrt{x}}$ = ______________________

d) $\lim_{x \to 0^+} x^{2x}$ = ______________________

e) $\lim_{x \to 0} \frac{\int_0^x e^{-t^2}\,dt}{x}$ = ______________________

f) $\lim_{x \to \infty} \tanh(4x)$ = ______________________

g) $\lim_{x \to \infty} \frac{\sqrt{e^{2x}+4x+10}}{e^x-1000x}$ = ______________________

4. (20 points)

a) Find the slope of the curve defined by $x^2 - y^2 + 2xy + 1 = 0$ at the point $(-2, 1)$.

b) Find the equation of the tangent line to the above curve at $(-2, 1)$.

c) Find $\frac{d^2y}{dx^2}$ at $(-2, 1)$.

5. (20 points)

The displacement x from the origin of a particle is given by the following function of time t :
(A negative displacement means a position to the left, a positive displacement is a position to the right.)
Distance is measured in **meters**, m, and time is measured in **seconds**, s.
$x(t) = 5\text{m} + 12\frac{\text{m}}{\text{s}}t - 3\frac{\text{m}}{\text{s}^2}t^2$; for $0\text{s} \leq t \leq 3\text{s}$

a) Determine the velocity, v , as a function of time. $v =$ ____________________

b) Determine the acceleration, a , as a function of time. $a =$ ____________________

c) What is the net displacement, Δx , from $t = 0\text{s}$ to $t = 3\text{s}$? $\Delta x =$ ____________________

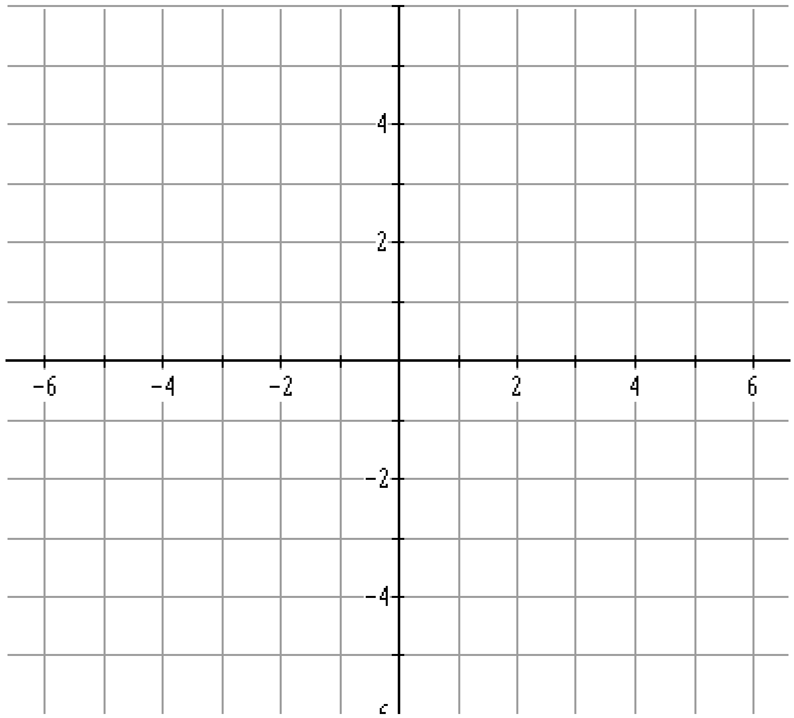
d) When is the particle moving to the left ?

e) When is the particle not moving?

f) When is the particle moving the fastest?

6. Solve the following differential equations and sketch the solution. (20 points)

a) $\frac{df}{dx} = -2xf(x)$ with $f(0) = 4$



7. (20 points)

Given $y = f(x) = xe^{-x^2}$

a) Compute $f'(x)$ $=$ ________________________________

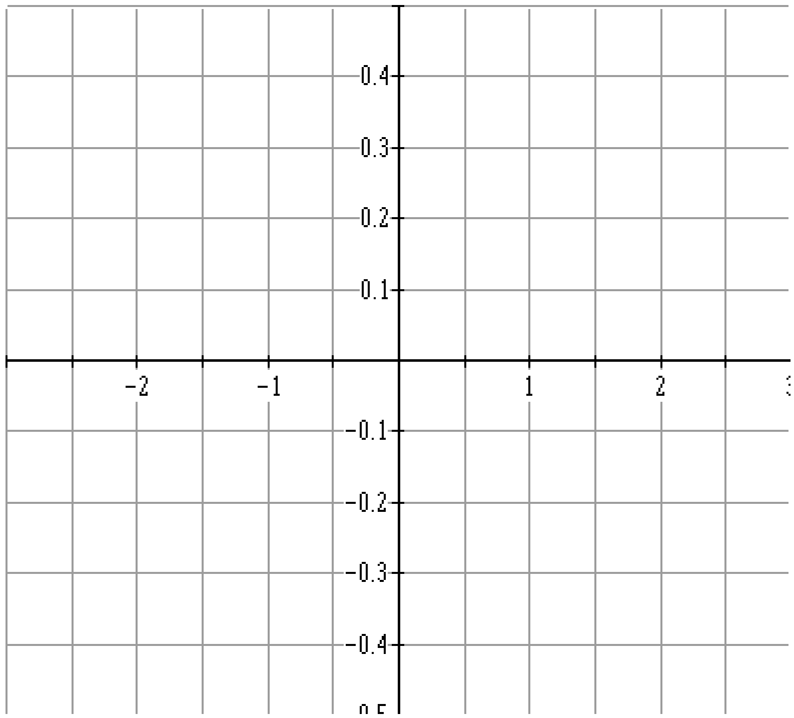
b) Compute $f''(x)$ $=$ ________________________________

c) Locate the roots of $f(x)$.

d) Find the location of all the maixma and minima of $f(x)$. Indicate which points are maxima (and whether they are absolute or only which are minima (absolute or local).

e) Find the location of all the inflection points of $f(x)$.

f) Sketch $f(x)$ on the next page.



8. (20 points)

a) Calculate the area of the region in the $x-y$ plane bounded by the curves $x=2$, $y=x^3$, and $y=0$.

b) Calculate the volume of the solid generated when the region in part a. is rotated about the x axis.

c) Calculate the volume of the solid generated when the region in part a. is rotated about the y axis.