Measurement

All measurements consist of two parts, a number part and a unit. For example, if we measure a table with a tape measure and report its width as 37.5 in , 37.5 is the number part, in is the unit part. The connection between the number part and unit part is the operation of multiplication. So when we write 37.5 in , we really mean (37.5)(1 in) , i.e., 37.5 multiples of the base unit of one inch.

There are many different kinds of things we can measure, but all common physical measurements can be reduced to just a few kinds of things. The four basic quantities are often listed as follows :

1. Length measured in units such as feet, miles, meters, etc.

2. Time measured in units such as seconds, minutes, hours, etc.

3. Mass (or weight) measured in units such as grams, kilograms, pounds, etc.

4. Electric charge measured in coulombs or amp-seconds.

All other measurements can be expressed as a combination of these. It should really be noted that mass and weight are not identical. Weight is the gravitational force acting on a mass and varies with position in space. For example, the weight of an object on the surface of the earth is about six times the weight of that same object on the surface of the moon. However, the mass of the object is the same in both locations. Despite this difference, in these notes we will not distinguish between mass and weight.

It is not sensible to add or subtract measurements of different kinds of things. For example, 15 lb + 7 ft is a meaningless operation. This is just the old adage that it's impossible to add apples and oranges! To add or subtract measurements requires the same kind of quantities, as in 9 ft + 8 ft = 17 ft . Note: we just add the numbers and carry the factor of the unit. This is just the distributive property discussed in Unit 3. Suppose we have 8ft + 36 in . Here, the quantities to be added are both lengths so the operation makes sense, but we can't actually perform the addition until we get the units to agree as in 8 ft + 36 in = 8 ft + 3ft = 11 ft . Here we "converted" 36 in to 3 ft . The operation could also have been done by converting 8 ft to 72 in , and adding 72 in + 36 in = 108 in .

While adding or subtracting different kinds of measurements is impossible, multiplying or dividing measurements is always possible. For example,

$5\text{ lb}\times 4\text{ ft} = 20\text{ ft}\cdot\text{lb}$, where a $\text{ft}\cdot\text{lb}$ is $1\text{ ft}\times 1\text{ lb}$ is a "foot pound", a measure of either energy or torque.

As a second example,

$100\text{ miles} \div 4\text{ gallons} = 25\text{ mpg (miles per gallon)}$, which measures fuel economy.

Measurement conversion is a necessary skill since the same set of units is often not used throughout a calculation. The basis of measurement conversion is the unit fraction. Any quantity Q remains unchanged when multiplied by 1.

$$Q = Q \cdot 1$$

The catch is that 1 has infinitely many "aliases". For example,

$$1 = \frac{12\text{ in}}{1\text{ ft}} = \frac{1\text{ ft}}{12\text{ in}} = \frac{4\text{ qt}}{1\text{ gal}} = \frac{1\text{ gal}}{4\text{ qt}} = \frac{60\text{ sec}}{1\text{ min}} = \frac{1\text{ min}}{60\text{ sec}} \quad .$$

All of these represent unit fractions, since the numerator is the same amount as the denominator. The trick is to use the "proper" aliases to cancel the units you don't want and get the units you do want. For example, to convert 18 in to ft we could use the following procedure.

$$18\text{ in} = \frac{18\text{ i}\not{\text{n}}}{1} \times \frac{1\text{ ft}}{12\text{ i}\not{\text{n}}} = \frac{18}{12}\text{ ft} = \frac{3}{2}\text{ ft} = 1.5\text{ ft} \quad .$$

Here the in unit was cancelled by appearing in both numerator and denominator.

Consider rounding 0.434 in to the nearest 64'th of an inch. The trick is to use the unit fraction

$$\frac{64}{64} \text{ as follows}: 0.434\text{ in} \times \frac{64}{64} = 0.434 \times 64 \times \frac{1\text{ in}}{64} = 27.776 \times \frac{1\text{ in}}{64} \approx \frac{28}{64}\text{ in} = \frac{7}{16}\text{ in} \quad .$$

So 0.434 in is seven sixteenth's of an inch to the nearest 64'th of an inch.

More complicated conversions can involve more than one unit fraction. The speed 100 feet per second can be converted to miles per hour correct to 1 decimal place by the following :

$$100\frac{\text{ft}}{\text{s}} = 100\frac{\text{ft}}{\text{s}} \times \frac{60\text{ s}}{1\text{ min}} \times \frac{60\text{ min}}{1\text{ hr}} \times \frac{1\text{ mile}}{5280\text{ ft}}$$

$$100\frac{\text{ft}}{\text{s}} = 100\frac{\not{\text{ft}}}{\not{\text{s}}} \times \frac{60\not{\text{s}}}{1\text{ m}\not{\text{i}}\text{n}} \times \frac{60\text{ m}\not{\text{i}}\text{n}}{1\text{ hr}} \times \frac{1\text{ mile}}{5280\not{\text{ft}}} = \frac{100 \times 60 \times 60}{5280}\frac{\text{mile}}{\text{hr}} = 68.2\text{ mph} \quad .$$

As an aide in setting up conversion calculations a set of equivalent measurements is presented on the following page.

This list also includes the formulas for converting temperature from Fahrenheit ($^\circ$F) to Celsius ($^\circ$C) . For example, to find the Fahrenheit equivalent of 40°C , we calculate as follows :

$$\text{Temp }^\circ\text{F} = 40 \times \frac{9^\circ\text{F}}{5} + 32^\circ\text{F} = 104^\circ\text{F} \quad ,$$

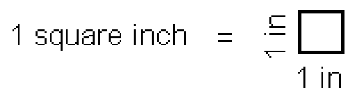
while the Celsius equivalent of minus 10°F is computed using the formula

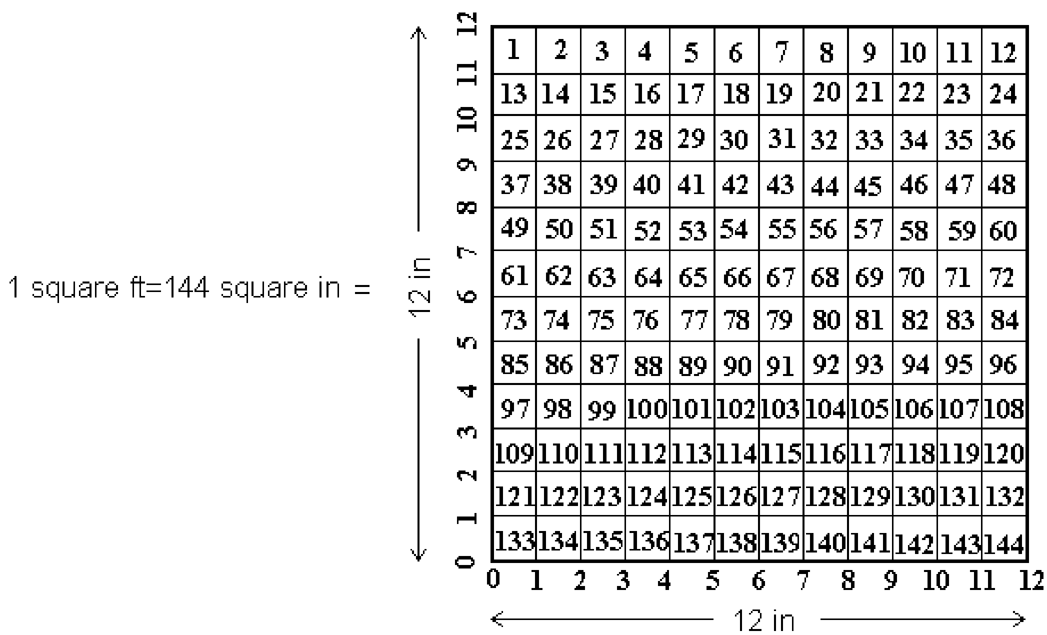
$$\text{Temp }^\circ\text{C} = (-10 - 32) \times \frac{5^\circ\text{C}}{9} = -23.3^\circ\text{C} \quad .$$

There is a definite relationship between length, area and volume measurements. Area is the amount of “two-dimensional space” inside of a planar figure. For example, a 2 ft by 6 ft rectangle has an area, A = (2 ft)(6 ft) = 12 ft^2. Here, the unit ft^2 is one square foot (sq ft) which literally means a one foot by one foot square. When we say that the area of the rectangle is 12 ft^2, we mean that we could fit exactly 12 one foot by one foot squares inside this rectangle.

Care must be taken when converting units of area. Suppose we want to calculate how many square inches are in an area of 1.6 ft^2. We need the unit fraction between square inches and square feet.

$$1\,\text{ft}^2 = (12\,\text{in})^2 = 12\,\text{in} \times 12\,\text{in} = 144\,\text{in}^2.$$





Conversion Relations for English and Metric Units

Linear Measure:

1 ft = 12 in
1 yd = 3 ft
1 mile = 5280 ft
1 rod = 16.5 ft
1 furlong = 220 yd
1 in = 2.54 cm
1 ft = 0.3048 m
1 yd = 0.9144 m
1 mile = 1.609344 km

Area Measure:

1 acre = 160 sq rods
1 sq mile = 640 acres
1 cm^2 = 0.15500031 in^2
1 m^2 = 1.195990046 yd^2
1 km^2 = 0.3861021585 sq mile

Volume Measure:

16 oz = 1 pt
2 pt = 1 qt
4 qt = 1 gal
1 gal = 0.13368056 ft^3
1 gal = 231 in^3
1 gal = 3.78541178 L
1 ft^3 = 7.48051948 gal
1 ft^3 = 28.31684659 L
1 L = 0.26417205 gal
1 L = 1.056688209 qt
1 L = 61.02374409 in^3
1 L = 0.001 m^3
1mL = 1 cm^3

Weight Measure:

16 oz = 1 lb
1 oz = 28.348 g
1 ton = 2000 lb
1 lb = 453.568 g
1 kg = 2.20474 lb

Temperature Conversions:

$$\text{Temp}°\text{F} = \text{Temp}°\text{C} \times \frac{9}{5} + 32°\text{F}$$

$$\text{Temp}°\text{C} = \left(\text{Temp}°\text{F} - 32\right) \times \frac{5}{9}$$

Time Measure:

1 min = 60 s
1 hr = 60 min
1 hr = 3600 s
1 day = 24 hours

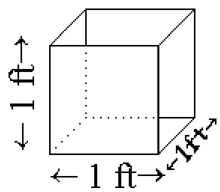
Note: when we evaluate $(12\text{ in})^2$ we square **both** the 12 **and** the in . This is illustrated below.

To perform the conversion,

$$1.6\text{ ft}^2 = 1.6\,\cancel{\text{ft}}^2 \times \frac{144\text{ in}^2}{1\,\cancel{\text{ft}}^2} = 1.6 \times 144\text{ in}^2 = 230.4\text{ in}^2.$$

Volume is the amount of "three-dimensional space" inside of a solid. For example, a 2 ft by 3 ft by 2 ft box has a volume, V = (2 ft)(3 ft)(2 ft) = 12 ft^3**.** Here the unit ft^3 is one cubic foot (cu ft), which literally means a one foot by one foot by one foot cube as shown below.

$1\text{ft}^3 = 1\text{ft} \times 1\text{ft} \times 1\text{ft} =$



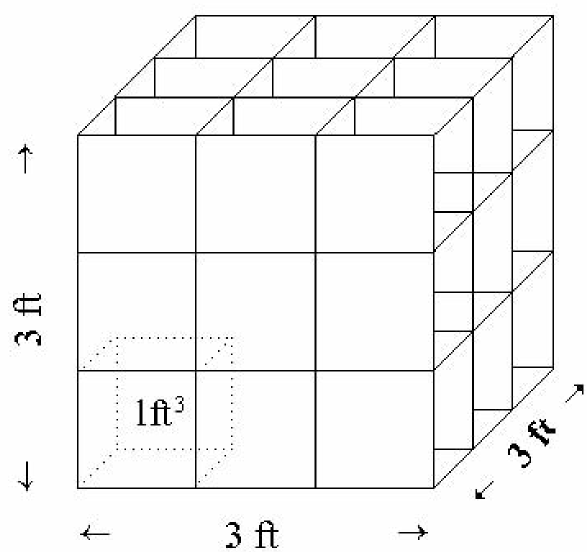
When we say that the volume of the box is 12 ft^3, we mean that we could fit exactly 12 one foot by one foot by one foot cubes inside this box. Like area conversions, volume conversions require careful setup. Suppose we wish to convert 12 ft^3 to cubic yards.

$$1\text{yard} = 3\text{ ft}$$

$$1\text{ cu yd} = 1\text{ yd}^3 = (3\text{ft})^3 = 3\text{ft} \times 3\text{ft} \times 3\text{ft} = 27\text{ ft}^3.$$

Note: when we evaluate $(3\text{ ft})^3$ we cube **both** the 3 **and** the ft . This is illustrated below.

$1\text{yd}^3 = 3\text{ft} \times 3\text{ft} \times 3\text{ft} = 27\text{ft}^3 =$



To perform the conversion,

$$12\text{ ft}^3 = 12\,\cancel{\text{ft}}^3 \times \frac{1\text{ yd}^3}{27\,\cancel{\text{ft}}^3} = \frac{12}{27}\text{yd}^3 = 0.44\text{yd}^3.$$

One of the consequences of the French Revolution of 1789 was the development of the metric system of measurement. This system was designed to replace the earlier French system, which like its English counterpart had its origins in medieval society and royal institutions. Three features make the metric system very attractive. First, it is built on powers of 10, just like our decimal number system. Every unit is a multiple of 10 of some other unit. Thus, "strange" English multipliers like 3, 12, and 16 are banished! Second, a deliberate effort was made to coordinate different measures. For example, the fundamental unit of volume, the liter symbolized by L, is simply related to the fundamental unit of length, the meter symbolized by m, through the equation 1 m^3 = 1000 L . Contrast this with the English system where 1 gal = 231 in^3 = 0.134 ft^3. The third advantage of the metric system is that it is "universal". It can be used with **any** kind of measurement in the same way. It is interesting to note, that the metric system was so well accepted and in place that when electrical measurements began some 150 years ago only metric units were developed and have survived. The customary electric units we are all know, the volt (V), amp (A), and ohm (Ω) are all metric. The metric system uses a two-part representation of all measurements. The first character or prefix indicates the power of 10 used, while the remainder of the measurement is the base unit. This is illustrated below.

10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}
G			M			k	h	da	base	d	c	m			μ			n

10^{-9} = 0.000000001
10^{-8} = 0.00000001
10^{-7} = 0.0000001
10^{-6} = 0.000001
10^{-5} = 0.00001
10^{-4} = 0.0001
10^{-3} = 0.001
10^{-2} = 0.01
10^{-1} = 0.1
10^0 = 1
10^1 = 10
10^2 = 100
10^3 = 1000
10^4 = 10,000
10^5 = 100,000
10^6 = 1,000,000
10^7 = 10,000,000
10^8 = 100,000,000
10^9 = 1,000,000,000

Prefix Names

G=giga
M=mega
k=kilo
h=hecto
da=deca
d=deci
c=centi
m=milli
μ=micro
n=nano

Important Metric Base Units

length - meter abbreviated m
time - second abbreviated s
mass - gram abbreviated g
volume - liter abbreviated L
(Note: the metric system has been set up so 1 mL= 1 cm^3)
power - watt - abbreviated w
energy - joule - j
force - newton -N
frequency - hertz abbreviated Hz
electric current - ampere abbreviated A
electric potential - volt abbreviated V
electric resistance - ohm abbreviated Ω
electric capacitance - farad abbreviated F
electric inductance - henry abbreviated H

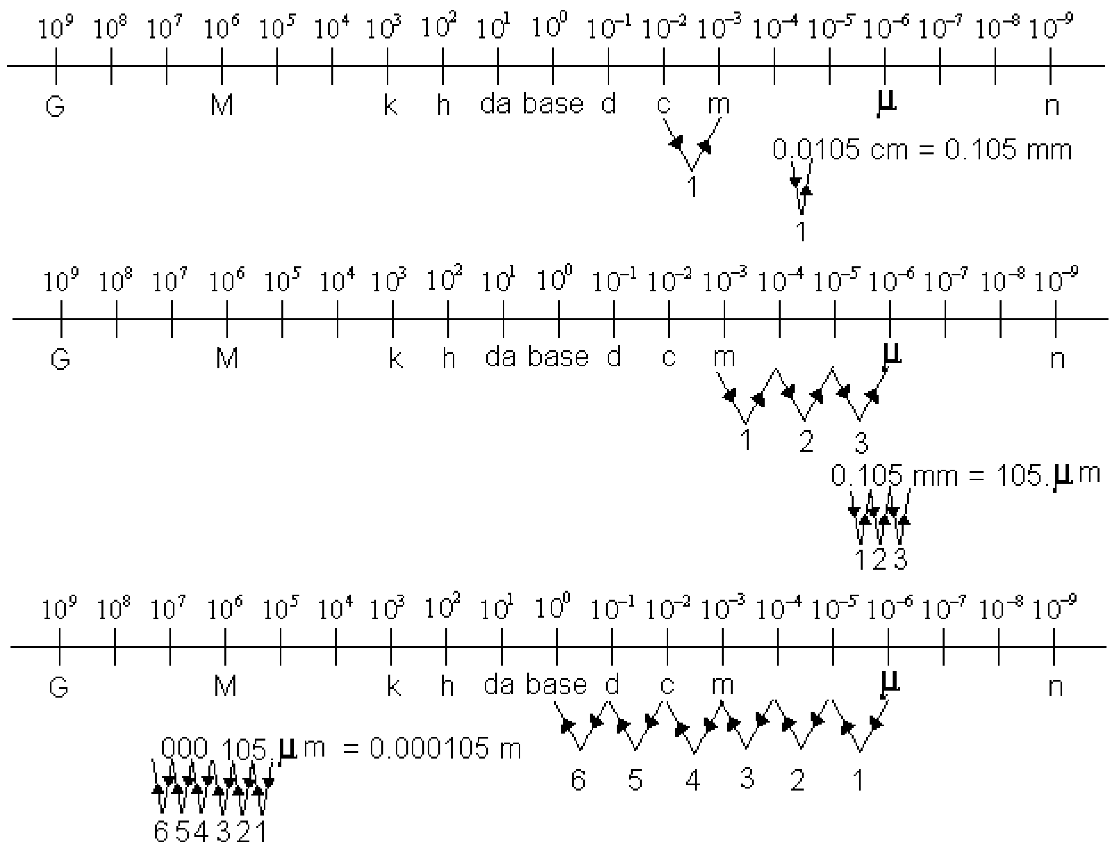
Conversions within the metric system are particularly easy. The steps are as follows:

1. Lay out a chart as shown below.

2. Locate the starting unit position and the final unit position on this chart and note the direction from the starting unit to the final unit.

3. Count the number of positions on the chart from the starting unit space to the final unit space.

4. Move the decimal point of the number part of the measurement the same number of decimal places as the count in Step 3 and in the same direction as noted in Step 2.



As before area and volume conversions within the metric system require careful setup. For example, suppose we want to convert 0.042 m^2 to square cm. The calculation can be setup as follows :

$$0.042\text{ m}^2 = 0.042\text{ m}^2 \times \left(\frac{100\text{ cm}}{1\text{ m}}\right)^2 = 0.042\text{ m}^2 \times \frac{1000\text{ cm}^2}{1\text{ m}^2} = 0.042 \times 10000\text{ cm}^2 = 420\text{ cm}^2.$$

As second example, to convert 187 mm^3 to mL, we proceed as shown below:

$$187\text{ mm}^3 = 187\text{ mm}^3 \times \left(\frac{1\text{ cm}}{10\text{ mm}}\right)^3 \times \frac{1\text{ mL}}{1\text{ cm}^3} = 187\text{ mm}^3 \times \frac{1\text{ cm}^3}{1000\text{ mm}^3} \times \frac{1\text{ mL}}{1\text{ cm}^3}$$

$$= \frac{187}{1000}\text{ mL} = 0.187\text{ mL}.$$

Conversions between metric and English units require conversion factors. For example, to convert 1.80 gallons per minute to m^3 per hour, we can use the following procedure:

$$1.80\,\frac{\text{gal}}{\text{min}} = 1.80\,\frac{\cancel{\text{gal}}}{\cancel{\text{min}}}\times\frac{60\,\cancel{\text{min}}}{1\,\text{hr}}\times\frac{3.7854\,\cancel{\text{L}}}{1\,\cancel{\text{gal}}}\times\frac{0.001\,\text{m}^3}{1\,\cancel{\text{L}}} = 1.80\times 60\times 3.7854\times 0.001\,\frac{\text{m}^3}{\text{hr}}$$

$$= 0.409\,\frac{\text{m}^3}{\text{hr}}.$$

Exercises:

Perform the following calculations with measurement numbers.

12 ft 3 in − 8 ft 8 in

1) ______________

$4.2\,\text{cm}\times 3.5\,\text{cm}^2$

2) ______________

$530\,\text{miles} \div 35\,\text{mpg}$ (mpg = miles per gallon)

3) ______________

Convert the following measurements as indicated. Write answers in the blank space provided. Round to one decimal place.

4. 185°F = ______________ °C

5. 17.6 liters = ______________ gal

6. 132 kg = ______________ lb

7. 127 ft^3 = ______________ yd^3

8. 129 mA = ______________ A

9. 0.589 m = ______________ mm

10. 58 cm = ______________ m

11. 127μV = ______________ V

12. 98 km/hour = ______________ mph

13. 10.0 °C = ______________ °F

14. 2.56 sq ft = ______________ cm^2

15. 2.49 gal = ______________ mL

16. 179cm^3 = ______________ in^3

Round to the nearest 32'nd of an inch : 0.165 in =

17) ______________

Round to the nearest 64'th of an inch : 0.645 in =

18) ______________

What size bolt, to the nearest 64'th of an inch, will fit a hole 12 mm in diameter?

19) ______________

A box has dimensions of 5 ft 6 in × 3 ft 9 in × 3 ft 4 in . How many cubic meters is this?

20) ______________

A car has a gas tank with a capacity of 50 L. If the car gets 33.5 miles per gallon, how many km can the car travel on a full tank?

21) ______________