**Intermediate Algebra 2 : Project 1 (Review)**

**/50**

**Name** _____

**Due 1/21/09**

Questions 1 through 30 are each worth 1 point. Questions 31 through 40 are each worth 2 points.
Evaluate the following.

1. $| 2-7 | - | 5-15 |$ = _____

2. For $x < 0$, $\frac{|x|}{x}$ = _____

3. For $x = 3$, $y = -2$, $\frac{xy^3-(xy)}{xy^2-x^2}$ = _____

Simplify the following.

4. $-4xy-6xy$ = _____

5. $-4xy(-6xy)$ = _____

6. $x[4x-3(5-2x)] - x(7x-10x)$ = _____

Solve the following equations for all real values which make them true.

7. $11x-9=24$

8. $3y+2-3(2y-5)=3(y-3)-2(3y-1)+20$

9. $6(s-2)-2-4(s-4)=3(4s+3)-(s-10)+10$

10. $|2x-1| = 7$

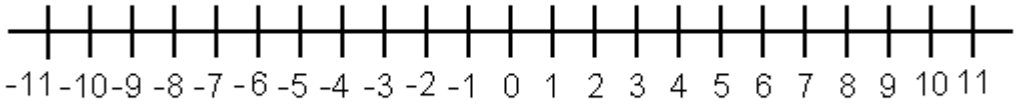
Rearrange the following formulas for the specified variable.

11. $2y = 3x + 8$ $y =$ _____

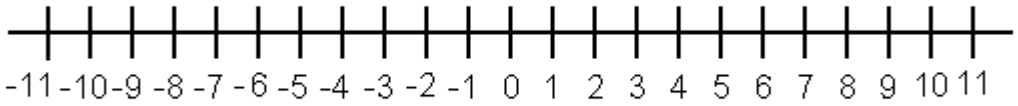
12. $6xy - 5x = 3(y - 2x)$ $y =$ _____

Solve and graph the solutions of the following inequalities.

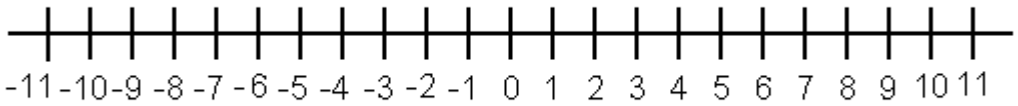
13. $2(x + 3) + 2(x + 1) \geq 6 - 2(x + 2)$




14. $|3x - 6| \geq 9$




15. $|4x - 6| \leq 10$




16. Which of the following relations define $y$ as a function of $x$ and which define $x$ as a function of $y$ ?

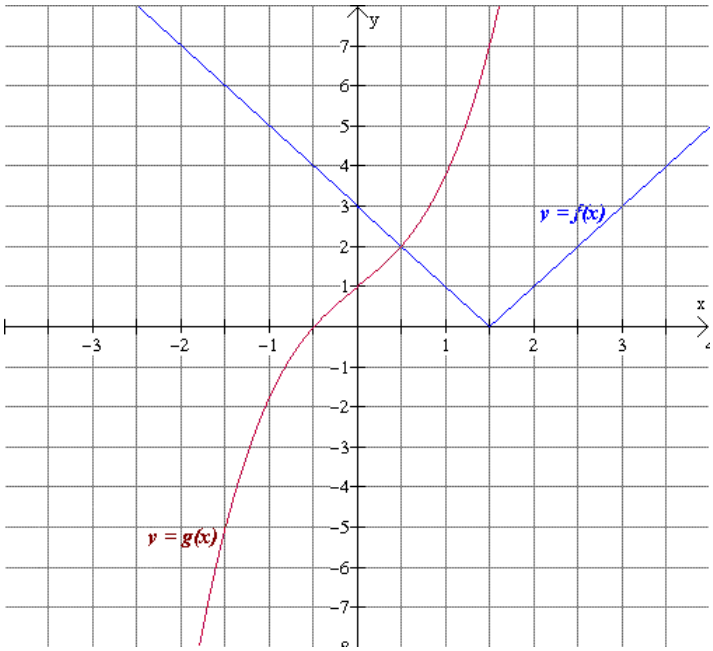
a) $x^2 + y^2 = 9$ b) $y = 2x^2 - 1$

c) $x + y = 4$ d) $x = 2y^2 - 1$

17. From the graphs of the functions, $f(x)$ & $g(x)$ , shown below **estimate** the following:

a) $f(0)\cdot g(0)$ = _____ b) $g\big(f(1.5)\big)$ = _____

c) $f\big(g(0)\big)$ = _____ d) $f\big(g(1)\big)$ = _____

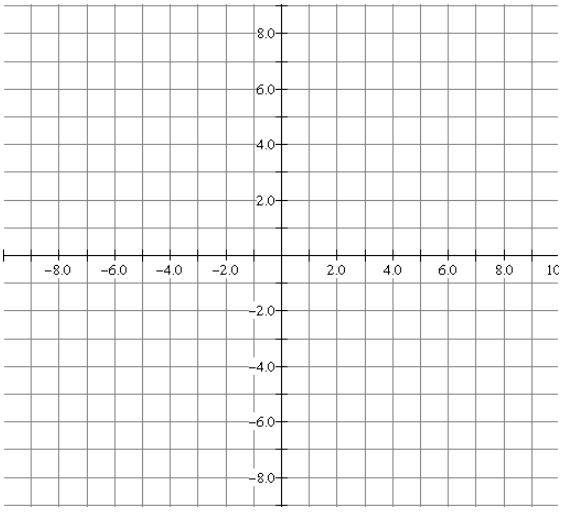



18. Graph the line $5x - 2y = 10$

$x$ intercept = _____

$y$ intercept = _____

slope = _____

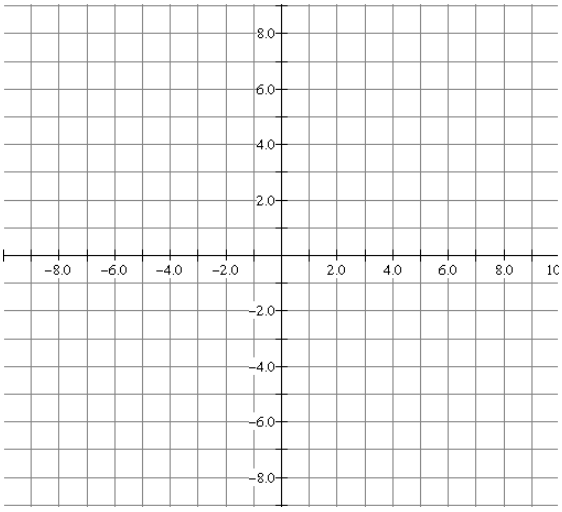


19. Graph the line $4x = 24$

$x$ intercept = _____

$y$ intercept = _____

slope = _____



20. Find the equation (in slope-intercept form) of the equation of the line which passes through $(-2, 7)$ and $(1, 1)$ .

21. Given $f(x) = 3x^2 - 2$ and $g(x) = x + 3$ , compute the following:

a) $f(2)$ = _____ b) $g(1)$ = _____

c) $f\big(g(1)\big)$ = _____ d) $g\big(f(2)\big)$ = _____

Perform the indicated operations and express the answer as a polynomial in standard form.

22. $(4x - 2)(4x + 5) - (7x - 2)(2x - 3)$ = _____

23. $(2x^2 - x + 3)(5x - 4)$ = _____

24. $(4a - 3b)^2$ = _____

25. $(4a - 3b)(4a + 3b)$ = _____

26. $(4a - 3b)(4a + 3b) - (4a - 3b)^2$ = _____

27. Perform the following polynomial long divisions.

$(b^7 - b^6 + 25b^4 - 8b^3 + 22b^2 + b - 10) \div (b^3 - 3b^2 + 7b + 5)$ = _____

Use synthetic division to find the quotient and remainder of the following divisions.

28. $\frac{3x^2+7x+9}{x+2}$ = _____

29. $\frac{3x^5-3x^4-x^3+2x^2+2x+4}{x+1}$ = _____

30. Given the polynomial, $f(x) = 3x^4 + 4x^3 - 6x^2 + 5x - 3$ , perform the requested evaluations using synthetic division.

a) $f(1)$ = _____ b) $f(2)$ = _____

Factor the following polynomials as completely as possible:

31. $6x^2 + 27x - 15$ = _____

32. $12xy + 8x - 9y - 6$ = _____

33. $16x^7y^5 - 25x^3y^3$ = _____

34. $40x^5z^3 + 135x^2y^6$ = _____

35. $x^2 - 2xy + y^2 - z^2$ = _____

36. $30x^2y^2z + 33xyz - 18z$ = _____

37. $8x^7 + 32x^6 + 32x^5$ = _____

38. $6x^3y^3 + 15(xy)^2z - 9xyz^2$ = _____

Solve the following equations.

39. $2x^2 + x = 6$

40. $20x^3 + 10x^2 - x = 5x^3 + 6x^2 + 2x$

 **Intermediate Algebra 2 : Project 2 (Chapter 6)** **/50**

**Name** _____ **Due 2/04/09**

Each problem is worth 2 points.

Simplify the following rational expressions :

1. $\frac{45x^2y^5z}{12(xyz)^3}$ = _____

2. $\frac{x^2+4x+4}{x^4-8x^2+16}$ = _____

3. $\frac{6x^2+12x+24}{x^3-8}$ = _____

Perform the following operations and simplify the result:

4. $\frac{p^2q^2}{2pq} \div \frac{-2pq}{p^3q^3}$ = _____

5. $\frac{6x^2-5x-6}{9x^2-4} \cdot \frac{2x^2+7x-15}{3x^2+10x-8}$ = _____

6. $\frac{q^2+2pq+p^2}{p^2-q^2} \div \frac{p+q}{p^2-2pq+q^2}$ = _____

7. $\frac{q^2-pq+p^2}{p^2-q^2} \div \frac{p^3+q^3}{q^2+2pq+p^2}$ = _____

8. $\frac{4x^2-5x+7}{x^3} - \frac{2x-3}{x^2} + \frac{9}{x}$ = _____

9. $\frac{y}{y-3} - \frac{2y^2}{2y^2-5y-3} + \frac{y-2}{2y+1}$ = _____

10. $\frac{1}{a+1} - \frac{a-1}{a^2+1} + \frac{a^3-a-2}{a^3+a^2+a+1}$ = _____

Express the following complex fraction as a simple rational expression in reduced form.

11. $\frac{\frac{5x-3}{3x-1}-3}{\frac{1}{3x-1}+1}$ = _____

12. $\dfrac{\frac{x+1}{x-1}-\frac{x-1}{x+1}}{\frac{1}{x}+\frac{1}{x^3-x}}$ = _____

13. Using WinPlot , generate and attach a computer plot of the following function. Use the window : $-5 \le x \le 5$ ; $-5 \le y \le 5$

$$y = f(x) = \frac{\frac{2}{x-2}+\frac{1}{x+1}}{\frac{3}{x-2}-\frac{3}{x^2-x-2}}$$

What value does $y$ appear to have? Verify this by simplification.

Solve the following equations for all real solutions.

14. $\frac{x}{x+2}+\frac{1}{x}=\frac{4}{x^2+2x}$

15. $\frac{1}{x}-\frac{3x}{x-2}=\frac{28-4x^2}{x^2-2x}$

16. $\frac{1}{x-2}-\frac{4}{x^2-4}=0$

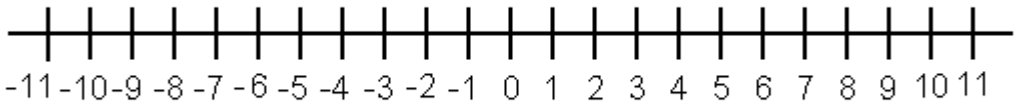
Solve the following literal equation for $x$ .

17. $a = \frac{2a-3cx}{2b+x} - c$

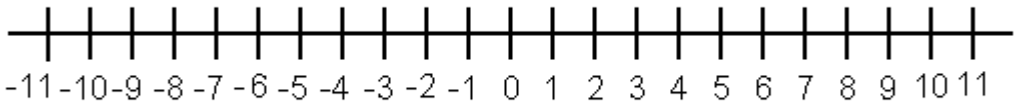
18. $\frac{1}{x} + \frac{1}{d} = \frac{1}{f}$

Solve and graph the following inequalities:

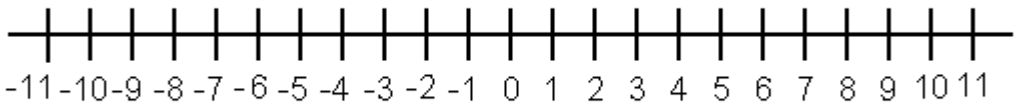
19. $\frac{3}{x-5} \geq \frac{x}{2}$




20. $\frac{3}{x-1} < \frac{x}{2}$




21. $\frac{3}{x} + 2 > \frac{2x}{x-3}$



22. A storage tank has two inlet pipes. The smaller pipe takes 12 minutes to fill the tank by itself. When both pipes are open the tank fills in 4 minutes 48 seconds. How long does it take the large pipe acting alone to drain the tank?

23. The pressure of a gas, $P$, varies directly as the temperature, $T$, and inversely as the volume, $V$. In the problems below give all results to three significant digits. If $P$ is $1.10$ atm (atmosphere) when $T = 300$ K and $V = 22.0\,l$, what is $V$ when $T = 450$ K and $P = 1.70$ atm ?

24. For fixed electrical charge, the capacitance of a pair of charged parallel circular plates varies directly as the square of the plate radius and inversely as the plate separation. If the capacitance is $34.7$ $\mu$F (micro farads) when the plate radius is $1.0$ cm and the plate separation is $4.0$ $\mu$m (microns, $1$ $\mu$m $= 0.0001$ cm), what is the plate separation, if for the same amount of charge and a plate radius of $2.0$ cm the capacitance is $347.0$ $\mu$F ?

25. The resistance, $R$, of a wire varies directly as the length, $l$, and inversely as the diameter, $d$, squared. If $R = 3.50\,\Omega$ when $l = 10.0$ m and $d = 0.125$ cm, what is the resistance of a wire made of the same material that is $5.00$ m long and has a diameter of $0.250$ cm ? (Give the answer to 3 significant digits.)

**Name** _____ **Due 2/24/09**

Problems 1 through 26 are each worth 1 point. Problems 27 through 38 are each worth 2 points.

Simplify the following using only positive rational exponents.

1. $(-2x^2)^2 \cdot (-2x^3y^2)^2$ = _____

2. $\left(\frac{-3x^0y^3}{2x^4y^2}\right)^3$ = _____

3. $-\left(-\left(\frac{-3x}{2y^3}\right)^2 \cdot \left(\frac{-2y}{3x}\right)^3\right)^3$ = _____

4. $\frac{(2x^{-3}y^{-2})^{-2} \cdot (\frac{1}{2}x^{-2}y^3)^{-3}}{x^3y^{-2}}$ = _____

5. $(a^{-1} - b^{-1})^{-1}$ = _____

6. $\left(-32\right)^{-\frac{3}{5}}$ = _____

7. $\left(\frac{36}{25}\right)^{-\frac{3}{2}}$ = _____

Express the following numbers in scientific notation as numbers between 1 and 10 times the appropriate power of 10.

8. $532,168$ = _____

9. $0.00000501 \times 10^{-3}$ = _____

10. $(0.00329) \cdot (2.5 \times 10^{-4})$ = _____

11. $\frac{(580{,}000{,}000) \cdot (1.25 \times 10^{-10})}{5.00 \times 10^{-4}}$ = _____

12. For $x > 0$, $\left(x^{-\frac{3}{2}}\right)^2 \cdot \left(4x^{-8}\right)^{-\frac{3}{2}}$ = _____

Express the following using only positive rational exponents.

13. $\sqrt[4]{a^4 + b^4}$ = _____

14. $\sqrt[5]{x^3}$ = _____

Express the following in radical notation reduced to simplest radical form with rationalized denominators.

15. $\left(-a^4b^3\right)^{\frac{1}{3}}$ = _____

16. $\left(x^5\right)^{-\frac{1}{2}}$ = _____

Simplify the following expressing all answers in simplest radical form with rationalized denominators. Assume for even roots that all variables represent positive numbers.

17. $\sqrt{72x^7}$ = _____

18. $\sqrt{\frac{8}{3x^3}}$ = _____

19. $\frac{1}{\sqrt[3]{-9x^{15}y^4}}$ = _____

20. $\frac{1}{\sqrt[5]{-9x^{15}y^4}}$ = _____

21. $\sqrt[8]{\frac{x^8}{16y^4}}$ = _____

22. $\sqrt[3]{-625} \cdot \sqrt[3]{\frac{-3}{5}}$ = _____

23. $5 + \frac{3}{\sqrt{2}+1} - \sqrt{18}$ = _____

24. $\frac{\sqrt{20}}{\sqrt{10}-\sqrt{5}}$ = _____

Simplify the following giving the correct answer for **any** real value of $x$

25. $\sqrt[5]{-x^5}$ = _____

26. $\sqrt{x^2}$ = _____

Simplify the following complex numbers, expressing the result in standard form $a + bi$ .

27. $5 + \sqrt{-25}$ = _____

28. $i^{297}$ = _____

29. $i^{-43}$ = _____

30. $\frac{4+\sqrt{-16}}{3-4i}$ = _____

31. $i\frac{(3-2i)^2-(3+i)(2+i)}{1+\sqrt{-1}}$ = _____

Solve the following equations for all real roots.

32. $\sqrt{2x+5} = 11$

33. $\sqrt{2x+5} = -3$

34. $\sqrt[3]{2x+5} = -3$

35. $6 - \sqrt{2x+5} = 3$

36. $\sqrt{x^2+9} = x+1$

37. $\sqrt{x+8} - \sqrt{2x-7} = 0$

38. Use Winplot and attach the computer plot or sketch the results below. Use the window : left = down = $-0.5$ ; right = up = $4$ . The function **sqrt** stands for the square root function. Check "lock interval" to fix the domain as stated. To generate the "correct" shape of the curves select Zoom/Square from the View menu.

| | Equa format | Domain | formula |
|---|---|---|---|
| Curve 1. | y=f(x) | low x = 0 high x = 4 | y = x^2 |
| Curve 2. | y=f(x) | low x = 0 high x = 4 | y = x |
| Curve 3. | x=f(t) | low t = 0 high t = 4 | x = t^2 ; y = t |
| Curve 4. | y=f(x) | low x = 0 high x = 4 | y = sqrt(x) |
| Curve 5. | x=f(t) | low t = 0 high t = 4 | x = sqrt(t) ; y = t |

a) What is true about the line $y = x$ (Curve 2) , with respect to Curve 1 and Curve 3?

b) How do the Curve 1 and Curve 5 compare? Explain why this is not surprising.

c) How do the Curve 3 and Curve 4 compare? Explain why this is not surprising.

**Name** _____ **Due 3/11/09**

Problem 1 through 28 are each worth 1.5 points. Problems 29 and 30 are each worth 4 points.

1. Solve the following equation by factoring.

$$12x^2 - 7x - 10 = 0$$

2. Solve the following equation by completing the square.

$$2x^2 - 8x + 26 = 0$$

3. Solve the following equation by completing the square.

$$3x^2 + 9x + 18 = 0$$

Solve the following equations for all real and/or complex roots by any method.

4. $4x^2 - 8x - 21 = 0$

5. $3x^2 - 12x = x^2 - 26$

6. $x^3 - 4x = 2x^2$

7. $\frac{10}{2x+1} = 4 - x$

8. $\frac{8}{2x^2+7x+3} + \frac{1}{2x+1} = 1$

Solve the following for all **real** roots.

9. $x = \sqrt{\frac{12-7x}{10}}$

10. $6 - \sqrt[3]{2x-1} = 3$

11. $\sqrt{x+3} - \sqrt{x-5} = 2$

12. $\sqrt{2x+1} - \sqrt{x-3} = 2$

13. $x - 2\sqrt{x} = 24$

14. $4x^4 + 100 = 41x^2$

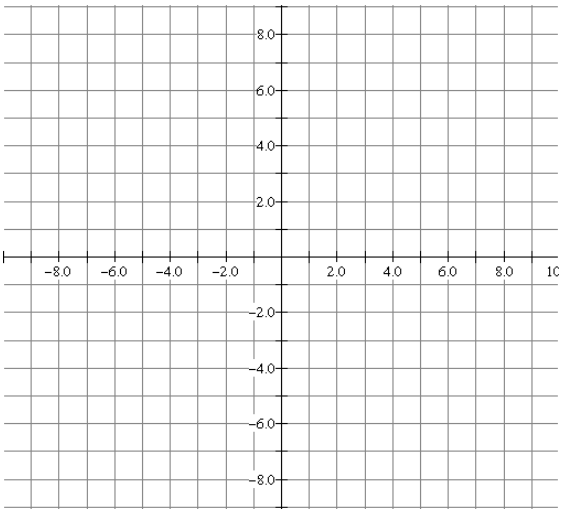
15. For what values of $c$ does the equation $2x^2 - 5x + c = 0$ have two distinct real roots?

16. A storage tank has two inlet pumps. The smaller pump takes $20$ minutes longer by itself to fill the tank than does the larger pipe. When both pumps are running the tank fills in $24$ minutes. How long does it take the large pump acting alone to fill the tank?

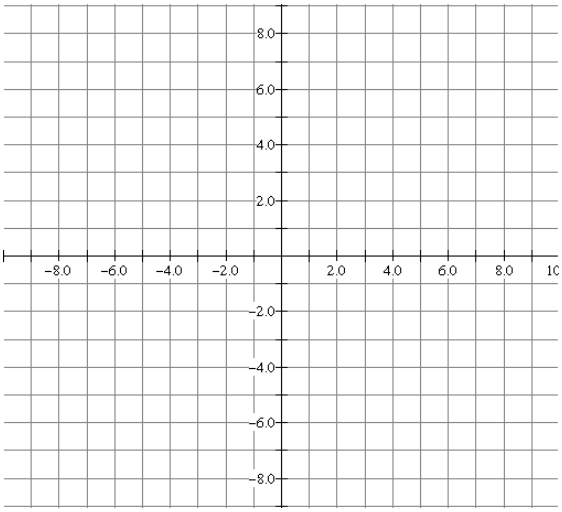
17. Mary and Steve each travel $420$ miles. Mary drove an average of $10$ mph faster than Steve and completed the trip in one hour's less time than Steve. What was each person's average speed?

Graph the following functions. If the graph is a parabola give the coordinates of the vertex and the equation of the axis of symmetry.

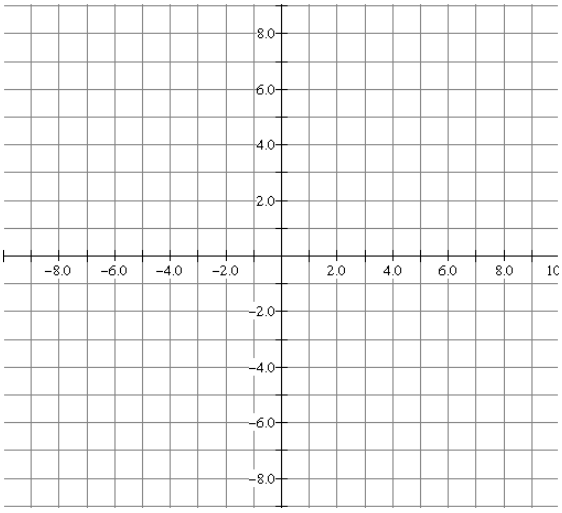
18. $y = f(x) = |2x - 4| + 4$



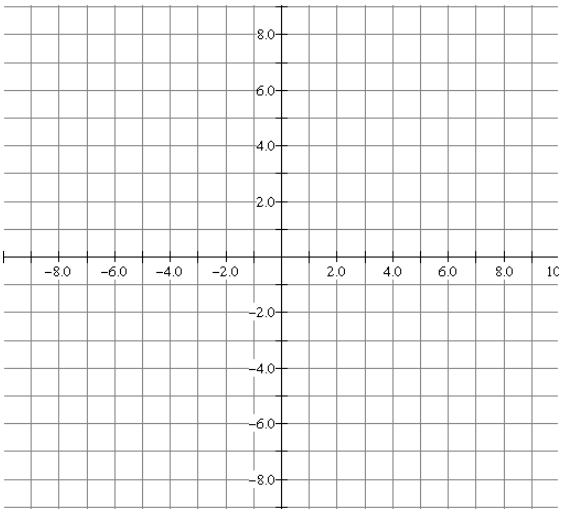
19. $y = f(x) = 6 - x^2$



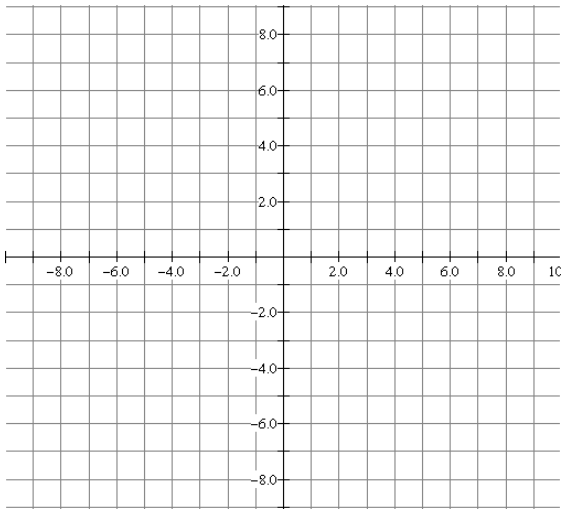

20. $y = f(x) = x^2 - 4x + 3$




21. $y = f(x) = 1 - 2x^2 + 6x$



22. $x = f(y) = -6 - 3y^2 - 12y$




Write an equation which generates each of the following curves.

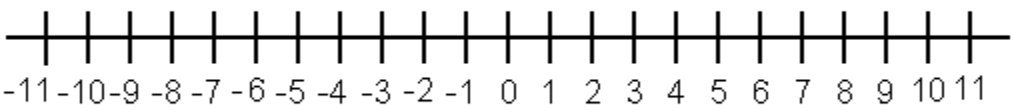
23. A circle of radius $5$ centered at $(-4, 2)$.

24. A parabola with vertex at $(-5, 3)$, axis of symmetry $x = -5$, and passes through the point $(1, -23)$.

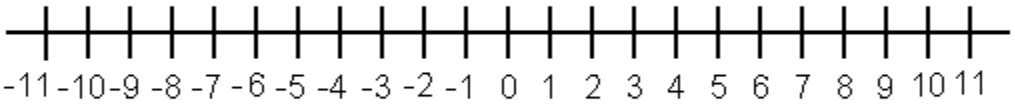
25. A parabola with vertex at $(2, -1)$, axis of symmetry $y = -1$, and passes through the point $(4, 0)$.

Solve and graph the solution set of the following inequalities.

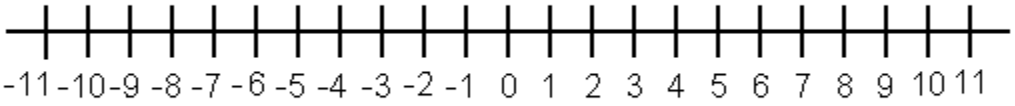
26. $x^2 - 2x \leq 15$



27. $4x^2 + x > 15 + 5x$




28. $\frac{3}{x-2} > \frac{x}{2}$




29. Using the WinPlot program graph the function $f(x) = 2x^2 - 3x - 6$ for $-5 \le x \le 5$.
Hand in the computer graph with your assignment. From your graph estimate the two roots of $f(x)$.

root #1 = _____

root #2 = _____

Now calculate 'exactly' the two roots of $f(x)$.

root #1 = _____

root #2 = _____

30. Using the WinPlot program graph the function $f(x) = x^3 - 2x^2 - 3x + 2$ for $-5 \le x \le 5$.
Hand in the computer graph with your assignment. From your graph estimate the three roots of $f(x)$.

root #1 = _____

root #2 = _____

root #3 = _____

**Name** _____ **Due 4/08/09**

Problems 3, 4, 5 and 10 are each worth 4 points. Problems 8 and 9 are each worth 3 points. All other problems are each worth 2 points.

1. Use WinPlot and attach the computer plot or sketch the results below. Use the window : left = down = $-5$ ; right = up = $5$ . To generate the "correct" shape of the curves select Zoom square from the View menu.

| | Equa format | Domain | formula |
|---|---|---|---|
| Curve 1. | y=f(x) | low x = $-5$ high x = $5$ | y = 10^x |
| Curve 2. | y=f(x) | low x = $-5$ high x = $5$ | y = x |
| Curve 3. | x=f(t) | low t = $-5$ high t = $5$ | x = 10^t ; y = t |
| Curve 4. | y=f(x) | low x = $-5$ high x = $5$ | y = log(x) |
| Curve 5. | x=f(t) | low t = $-5$ high t = $5$ | x = log(t) ; y = t |

a) What is true about the line $y = x$ , with respect to the Curve 1 and Curve 3?

b) How do the Curve 1 and Curve 5 compare? Explain why this is not surprising.

c) How do the Curve 3 and Curve 4 compare? Explain why this is not surprising.

2. Use WinPlot and attach the computer plot or sketch the results below. Use the window : left = down = $-5$ ; right = up = $5$ . To generate the "correct" shape of the curves select Zoom square from the View menu.

| | Equa format | Domain | formula |
|---|---|---|---|
| Curve 1. | y=f(x) | low x = $-5$ high x = $5$ | y = e^x |
| Curve 2. | y=f(x) | low x = $-5$ high x = $5$ | y = x |
| Curve 3. | x=f(t) | low t = $-5$ high t = $5$ | x = e^t ; y = t |
| Curve 4. | y=f(x) | low x = $-5$ high x = $5$ | y = ln(x) |
| Curve 5. | x=f(t) | low t = $-5$ high t = $5$ | x = ln(t) ; y = t |

a) What is true about the line $y = x$ , with respect to the Curve 1 and Curve 3?

b) How do the Curve 1 and Curve 5 compare? Explain why this is not surprising.

c) How do the Curve 3 and Curve 4 compare? Explain why this is not surprising.

3. Given $f(x) = 2x + 1$ and $g(x) = \frac{1}{x+2}$

a) What is the domain of $f$?

b) What is the range of $f$ ?

c) What is the domain of $g$ ?

d) What is the range of $g$ ?

e) $f(x) \cdot g(x)$ = _____

f) $(g \circ f)(x)$ = _____

g) $(f \circ g)(x)$ = _____

4. Using the functions $f(x)$ and $g(x)$ defined in problem 3, find the inverse functions

a) $f^{-1}(x)$ = _____

b) $g^{-1}(x)$ = _____

c) $f^{-1}\big(f(x)\big)$ = _____

d) $g^{-1}\big(g(x)\big)$ = _____

5. For the functions $f(x)$ and $g(x)$ defined in problem 3 and the inverse functions of problem 4,

a) What is the domain of $f^{-1}$ ?

b) What is the range of $f^{-1}$ ?

c) What is the domain of $g^{-1}$ ?

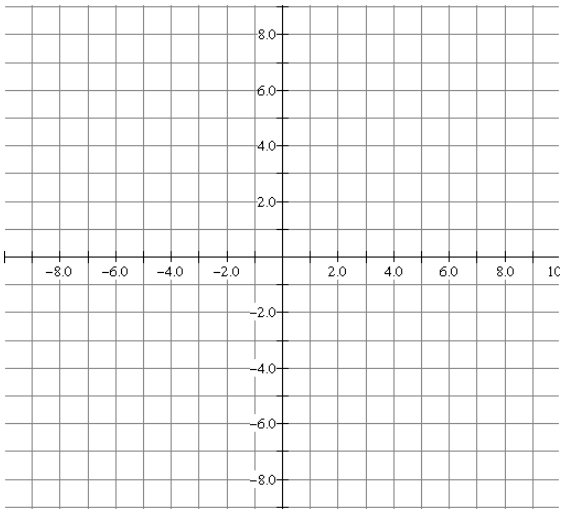
d) What is the range of $g^{-1}$ ?

e) $f\big(f^{-1}(x)\big)$ = _____

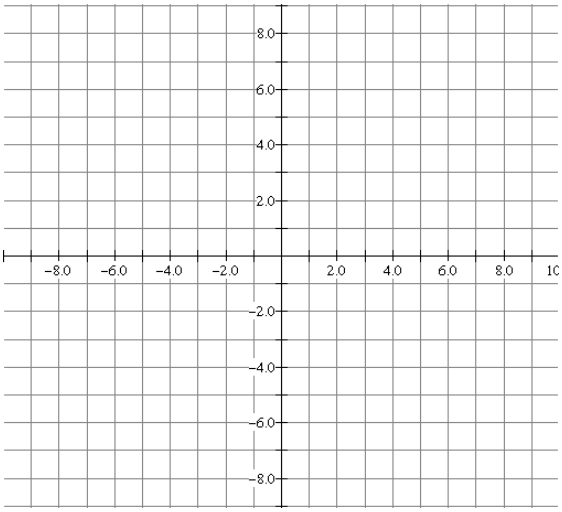
f) $g\big(g^{-1}(x)\big)$ = _____

For the following functions : i) State the domain ii) State the range iii) Sketch the curve

6. $y = f(x) = 2^{-x}$



7. $y = f(x) = \log_4(x)$




8. Change the following expressions from exponential form to logarithmic form:

a) $10^9 = 1,000,000,000$

b) $2^{10} = 1024$

c) $A = e^r$

d) $2^{-3} = 0.125$

e) $4^{-\frac{1}{2}} = 0.50$

f) $a^b = c$

9. Change the following from logarithmic to exponential form:

a) $\log_5(125) = 3$

b) $\log_8(0.125) = -1$

c) $\log_4(2) = \frac{1}{2}$

d) $\log_x(m) = n$

10. Compute the following:

a) $\log_5(0.20)$ = _____

b) $\log_4(16)$ = _____

c) $\log_2(64)$ = _____

d) $\log_x(x^p)$ = _____

e) $e^{\ln(3x)}$ = _____

f) $\log(1000)$ = _____

g) $\ln(1000)$ = _____

h) $\log_5(1000)$ = _____

Solve the following equations for all real roots. If no real solution exists, write 'No Solution' . You may leave answers in a form which involves an irrational base-10 or natural logarithm. For example, $y = \frac{5}{\ln(3)}$ , or you may give the numerical answer $y = 4.551196133...$ . Each problem is worth 1 point.

11. $a^5 = 32$ $a =$ _____

12. $\log_r(27) = 3$ $r =$ _____

13. $10^x = 10,000$ $x =$ _____

14. $10^z = 50,000$ $z =$ _____

15. $\log_4(y) = 5$ $y =$ _____

16. $\ln(x) = 2$ $x =$ _____

17. $\ln(x) = -2$ $x =$ _____

18. $\ln(y^5) = 5$ $y =$ _____

19. The decay of a radioactive material is expressed by the function $A = f(t) = A_0(2)^{-\frac{t}{T}}$ , where $A$ is the amount of material left after a time $t$ has elapsed, $A_0$ is the starting amount of material, and $T$ is the half-life of the material. If in $1500$ years time, $20\%$ of the material decays (leaving $80\%$ of the material still present), what is the half-life?

20. Four thousand dollars is deposited in a savings account with a fixed yearly interest rate of $3.75\%$.
a) After five years how much money is in the account if the it is compounded quarterly?

b) After five years how much money is in the account if the it is compounded continuously?

c) In this situation how much money does continuous compounding gain versus quarterly compounding?

d) If the account is compounded quarterly, how many years would it take for the amount to reach $\$6000.00$ ?

# Instructions for the Group Lab: The Simple Pendulum

The following guidelines should be adhered to in forming your lab group, performing the experiments, and writing up the labs.

**Group Requirements:**

Each group must consist of at least two individuals but no more than four individuals. You are free to form your own groups, but if you can't find a partner see me and I'll assign you to a group. Two class periods will be devoted to doing each lab, but probably some of the report writing will have to be done outside of class. It is up to the group to decide any internal division of labor, eg., who is responsible for data observation and or recording, who will do the algebra, who will check the work, who will write up the what parts of the report. It is possible that in one group a single individual writes the entire report, while in another group everyone writes up a different part. It is in your own best interest to insist that you understand the entire lab report. You are free to use any written resources or computing technology in doing your analysis.

**Report Requirements:**

Each group must hand in **one** report for a given lab which should include the following :

1. The names of all group participants. If the report writers feel an individual did not perform his/her assigned task, you are free to delete that person's name from the report. I will arbitrate all appeals on such disagreements and reserve the right to give either a written or oral exam to decide the issue.

2. The conclusions stated neatly in sentences which are both concise and complete.

3. The work attached in a way which is both neat and clear. Answers should be presented in the same order as the associated questions .

**Grading:**

1. Each person in the group will receive the same point total out of 40 that the lab report receives. Appeals on this are permitted, but I reserve the right to then administer either an oral or written exam to such an individual to replace the group score. Thus, it is the responsibility of everyone in the group to review the analysis, conclusions and answers to all of the questions.

2. Grades will be based on both the quality of the data taken and the correctness of the methods used to analyze the data. Thus, a correct conclusion arrived at by accident using faulty mathematics will not count for much. Points will be deducted for incomplete, illegible, sloppy or incomprehensible answers.

**Lab Scheduled 4/13/09 & 4/14/09** **Lab Report : Due 4/23/09**

**/50**

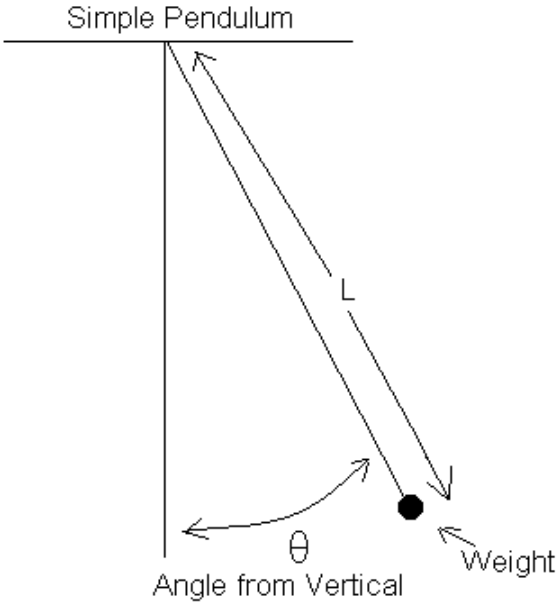
**Name _____** **Name _____**

**Name _____** **Name _____**

**Purpose :** To investigate the relationship between the length of a simple pendulum and the time it takes to complete a full swing.

**Equipment :** String, stop watch, weights, meter stick, protractor, ( a balance if available).

**General Procedure :** Tie one of the weights to the end of the string. From the center of the weight measure off the specified length $L$ of the string. Holding the string at this distance, let the lead weight swing freely from an initial position that makes a $20°$ angle ($\theta = 20°$) with the vertical . Measure the time for $10$ full swings of the weight. Divide this time by $10$ to obtain the period, $T$, the time for one full swing. Repeat this procedure for each of the specified values of $L$. Then repeat the experiment for $\theta = 25°$ and $\theta = 30°$. Finally, pick a second, different mass weight and repeat the entire set of measurements.




**I. Data Collection** (16 points)

**Data Table for First Weight ( If Balance available mass of weight = _____ )**

| | $\theta = 20°$ | | $\theta = 25°$ | | $\theta = 30°$ | |
|---|---|---|---|---|---|---|
| $L$ | **Time for 10 Swings** | **Period $T$** | **Time for 10 Swings** | **Period $T$** | **Time for 10 Swings** | **Period $T$** |
| 10.0 cm | | | | | | |
| 15.0 cm | | | | | | |
| 20.0 cm | | | | | | |
| 25.0 cm | | | | | | |
| 30.0 cm | | | | | | |
| 35.0 cm | | | | | | |
| 40.0 cm | | | | | | |
| 45.0 cm | | | | | | |
| 50.0 cm | | | | | | |
| 55.0 cm | | | | | | |
| 60.0 cm | | | | | | |
| 65.0 cm | | | | | | |
| 70.0 cm | | | | | | |
| 75.0 cm | | | | | | |
| 80.0 cm | | | | | | |

**Data Table for Second Weight ( If Balance available mass of weight = _____ )**

| | $\theta = 20°$ | | $\theta = 25°$ | | $\theta = 30°$ | |
|---|---|---|---|---|---|---|
| $L$ | **Time for 10 Swings** | **Period $T$** | **Time for 10 Swings** | **Period $T$** | **Time for 10 Swings** | **Period $T$** |
| 10.0 cm | | | | | | |
| 15.0 cm | | | | | | |
| 20.0 cm | | | | | | |
| 25.0 cm | | | | | | |
| 30.0 cm | | | | | | |
| 35.0 cm | | | | | | |
| 40.0 cm | | | | | | |
| 45.0 cm | | | | | | |
| 50.0 cm | | | | | | |
| 55.0 cm | | | | | | |
| 60.0 cm | | | | | | |
| 65.0 cm | | | | | | |
| 70.0 cm | | | | | | |
| 75.0 cm | | | | | | |
| 80.0 cm | | | | | | |

Give a brief but accurate description of the procedure **you** followed in obtaining your data. Use diagrams where necessary and identify all pertinent variables.

**II. Data Analysis** (16 points) ( Feel free to use a **spreadsheet** to perform the required calculations and plot the graphs. )

What were the relevant variables in this experiment?

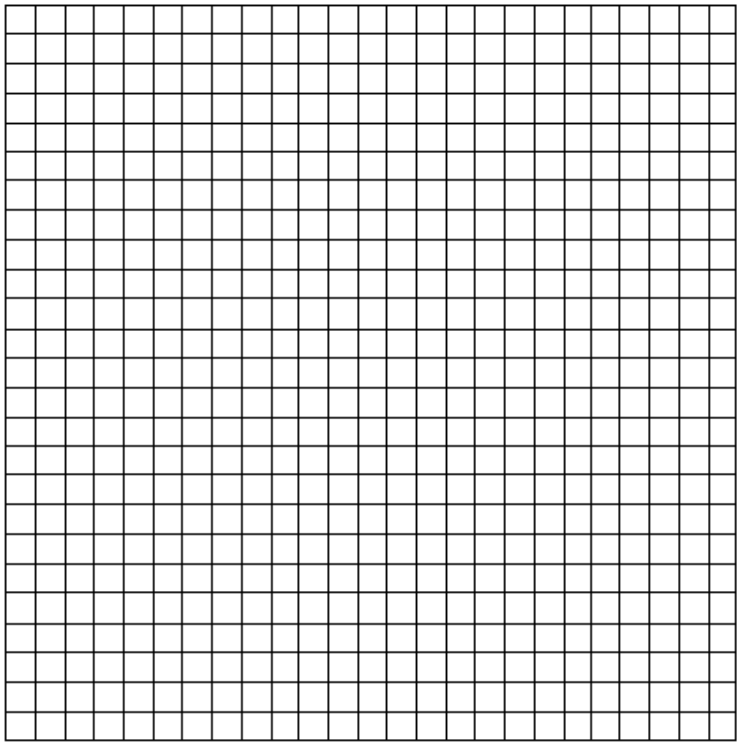
Which variables were independent and which were dependent?

Construct a graph of $T$ versus $L$ . **Label** all axes and label each curve as to the weight and angle $(\theta)$ used. You may if you wish put all six curves on the same graph. You may use either the grid provided or your own graph paper.

In general, how did the period depend on the initial angle $\theta$ ?

In general, how did the period depend on the weight used ?

## Graphs of $T$ versus $L$

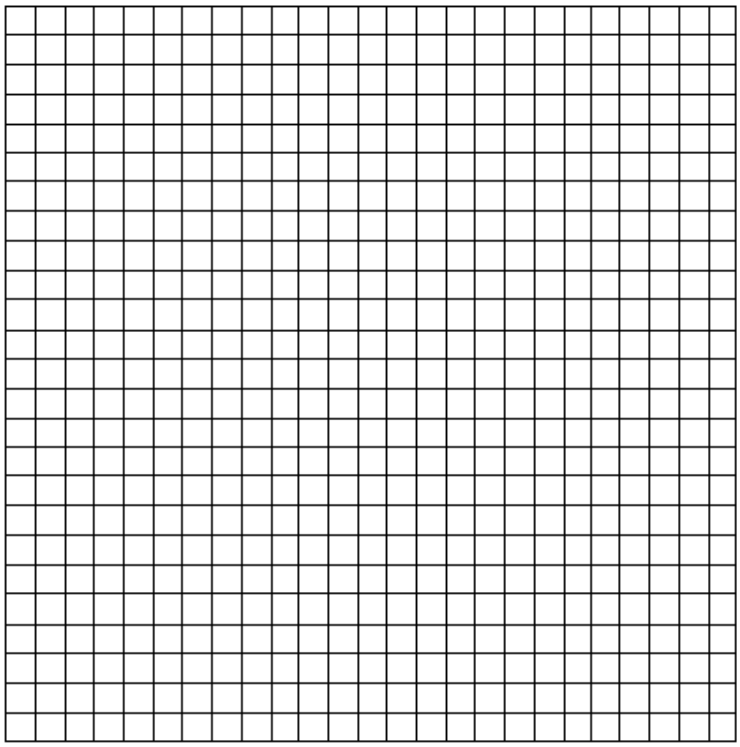


Construct a graph of $\ln(T)$ versus $\ln(L)$. **Label** all axes and label each curve as to the weight and angle $(\theta)$ used. You may if you wish put all six curves on the same graph. You may use either the grid provided or your own graph paper. Record the data for these graphs in the table provided.

## Ln($L$) and Ln($T$) Data

| | **Ln($L$) and Ln($T$) Data for First Weight** | | | | **Ln($T$) Data for Second Weight** | | |
|---|---|---|---|---|---|---|---|
| | | **Ln($T$)** | | | **Ln($T$)** | | |
| $L$ | **ln($L$)** | $\theta = 20°$ | $\theta = 25°$ | $\theta = 30°$ | $\theta = 20°$ | $\theta = 25°$ | $\theta = 30°$ |
| 10.0 cm | | | | | | | |
| 15.0 cm | | | | | | | |
| 20.0 cm | | | | | | | |
| 25.0 cm | | | | | | | |
| 30.0 cm | | | | | | | |
| 35.0 cm | | | | | | | |
| 40.0 cm | | | | | | | |
| 45.0 cm | | | | | | | |
| 50.0 cm | | | | | | | |
| 55.0 cm | | | | | | | |
| 60.0 cm | | | | | | | |
| 65.0 cm | | | | | | | |
| 70.0 cm | | | | | | | |
| 75.0 cm | | | | | | | |
| 80.0 cm | | | | | | | |

**Graphs of $\ln(T)$ versus $\ln(L)$**



**III. Interpretation** (14 points)
Is the relationship between $T$ and $L$ linear? Explain your answer.

Is the relationship between $\ln(T)$ and $\ln(L)$ linear? Explain your answer.

From elementary physics the period of a simple pendulum for small initial angles satisfies a 'power law' relationship to the length. That is $T = A \cdot L^{p}$. Where $A$ is a constant independent of $L$. From your data obtain an estimates of $p$ and $A$. Explain how you obtained these estimates.

**Hint:** The"best" or regression fit of a straight line through a set of $n$ data points $(x_i, y_i)$ is given by the linear model $y = mx + b$, where $m$ is the slope and $b$ is the $y$ intercept. The slope of the regression line is given by
$m = \frac{n\sum x_i y_i - \sum x_i \cdot \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$ while the $y$ intercept can be calculated as $b = \frac{\sum x_i^2 \sum y_i - \sum x_i \cdot \sum x_i y_i}{n\sum x_i^2 - (\sum x_i)^2}$ .
Here we are using **summation notation** where $\sum x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + ... + x_n y_n$ ,
$\sum x_i = x_1 + x_2 + x_3 + ... + x_n$, $\sum y_i = y_1 + y_2 + y_3 + ... + y_n$ ,and $\sum x_i^2 = x_1^2 + x_2^2 + x_3^2 + ... + x_n^2$ .

Estimated value of $p =$ _____ Estimated value of $A =$ _____

For the same weight you used estimate the period if $L = 100$ cm . If you have the equipment you may wish to check this estimate by actually measuring the period.

Estimated period for a length of $100$ cm $=$ _____

**IV. Application** (4 points)
Below is a table of data on the electrical resistance, $R$ , for a 1 meter length of different gauge copper wire. The wire's diameter is $d$ .

**Copper Wire Resistance Data**

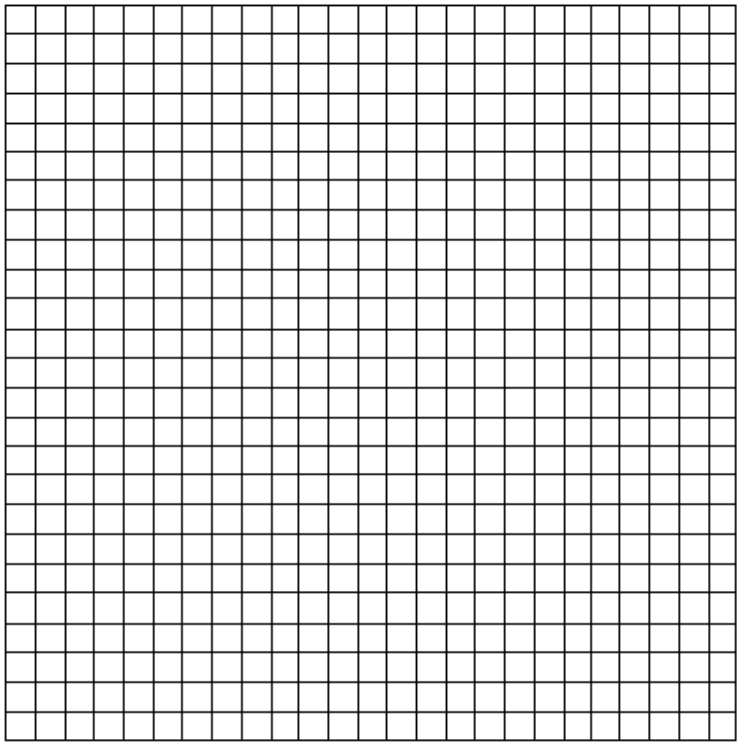
| Gauge | $d$ (cm) | $R$ (m$\Omega$) |
|---|---|---|
| **30** | 0.03175 | 223.7 |
| **25** | 0.05556 | 73.0 |
| **20** | 0.09525 | 24.9 |
| **15** | 0.1786 | 7.1 |
| **12** | 0.2778 | 2.9 |
| **10** | 0.3572 | 1.8 |
| **9** | 0.3969 | 1.4 |
| **8** | 0.4366 | 1.2 |
| **7** | 0.4763 | 1.0 |
| **6** | 0.5159 | 0.8 |
| **5** | 0.5556 | 0.7 |
| **4** | 0.5953 | 0.6 |

According to theory the resistance is related to diameter by a 'power law' . That is $R = A \cdot d^{p}$ , where $A$ is a constant independent of $d$. From the table obtain an estimate of $p$. Explain how you obtained this estimate.

Estimated value of $p =$ _____ Estimated value of $A =$ _____

Estimate the resistance for a 1 meter length of 35 gauge copper wire ($d = 0.01984$ cm) .

Estimated resistance for a 1 m length of 35 gauge copper wire $=$ _____



**Name** _____ **Due 5/04/09**

Each problem is worth 2 points.

1. Let $\mathcal{L}$ be the line that passes through $(-3, 5)$ and is parallel to the line $5x - 4y = -18$.
Let $\mathcal{M}$ be the line that passes through $(-3, 5)$ and is perpendicular to the line $5x - 4y = -18$.

Equation of $\mathcal{L}$ in slope-intercept form : _____

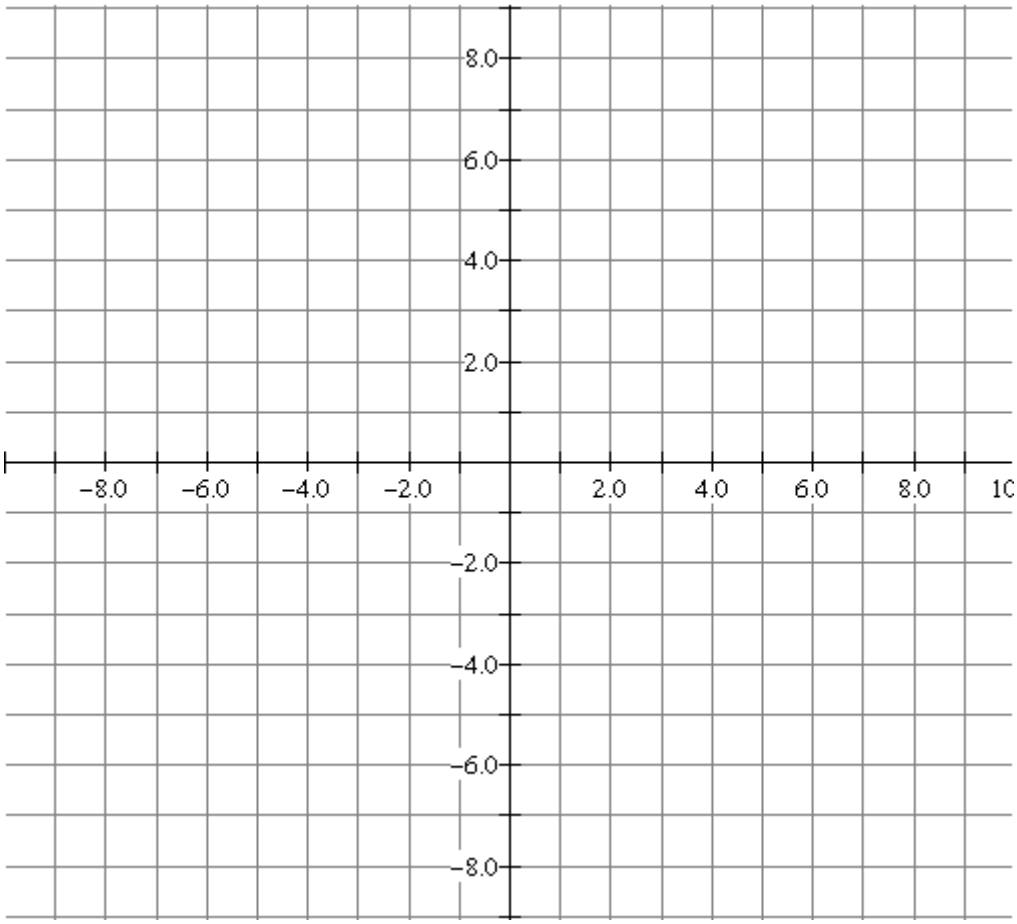
Equation of $\mathcal{M}$ in slope-intercept form : _____

The $x$ intercept of $\mathcal{L}$ = _____ The $x$ intercept of $\mathcal{M}$ = _____

The $y$ intercept of $\mathcal{L}$ = _____ The $y$ intercept of $\mathcal{M}$ = _____

The slope of $\mathcal{L}$ = _____ The slope of $\mathcal{M}$ = _____

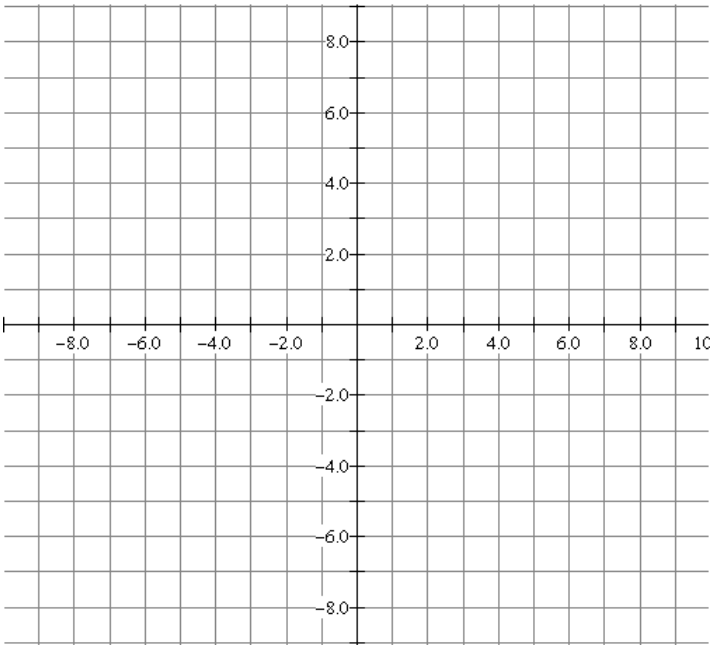
Graph the lines $\mathcal{L}$ and $\mathcal{M}$



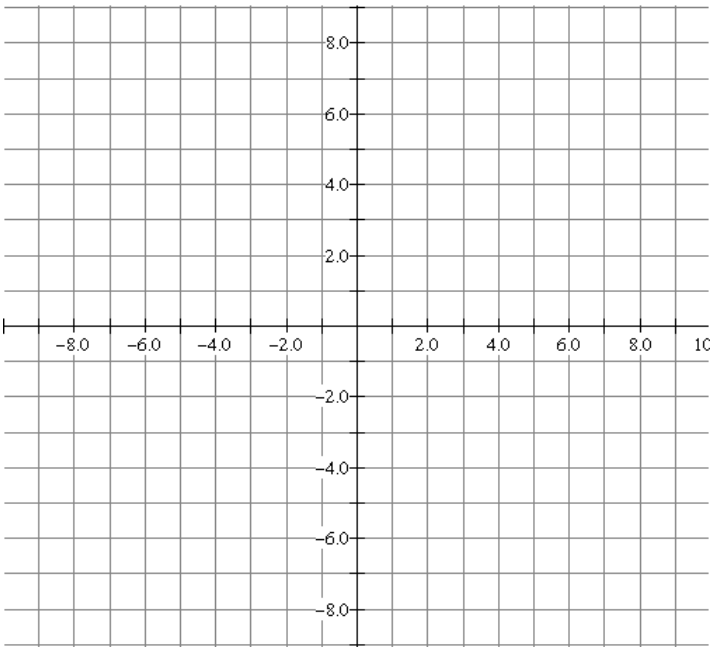
2. Plot each line to find the solution of the system of equations:

$$\begin{array}{rcrcr} x & + & 3y & = & -7 \\ 2x & - & y & = & 7 \end{array}$$

Then check your answer by solving the system algebraically.




3. Graph the solution set of the inequality : $-4y + 6x < 24$




4. Using the WinPlot program . graph the two linear functions $y = f(x) = 2x + 3$ and $y = f(x) = -2x + 1$ from low x $= -5$ to high x $= 5$ . Hand in the computer graph with your assignment. From the graph estimate the point of intersection of the two lines. Then algebraically solve the system of equations which corresponds to this intersection.

Point of intersection = _____

Solve the following systems of equations algebraically.

5.
$$\begin{array}{rcrcr} 3x & + & 2y & = & 5 \\ 2x & - & y & = & -6 \end{array}$$

6.
$$\begin{array}{rcrcr} x & - & 3y & = & -2 \\ 2x & - & 6y & = & 5 \end{array}$$

7.
$$\begin{array}{rcrcr} x & - & 3y & = & -7 \\ -2x & + & 6y & = & 14 \end{array}$$

8. A man travels $620$ miles by both train and bus. The bus averages $60$ mph, while the train averages $40$ mph. The total travel time on both train and bus was $12$ hours. How long did the bus trip last?

9. How many grams of pure tin must be mixed with $200$ g of $25\%$ tin solder and $100$ g of $30\%$ tin solder in order to have a final solder which is $40\%$ tin?

10. Mary delivers one less package than twice as many as Terri. Together Terri and Kevin deliver five more packages than Mary. All together the three of them deliver fifty-one packages. How many packages are delivered by each person?

11. Joan has $\$1.76$ in change. She has four less dimes than pennies, but one more dime than twice the number of quarters. She also has one more nickle than quarters. How many of each kind of coin does Joan have?

Solve and classify (consistent and independent, inconsistent, or dependent) the following systems of equations .
If the solutions are dependent, give the linear "form" of the infinite number of solutions.

12.
$$\begin{array}{rcrcrcr} 4x & + & 3y & = & -5 \\ 3x & - & y & = & -7 \end{array}$$

13.
$$\begin{array}{rcrcrcr} x & + & y & + & 2z & = & -5 \\ 2x & - & y & + & z & = & -7 \\ 3x & + & 2y & - & 3z & = & 2 \end{array}$$

14.
$$\begin{array}{rcrcrcr} x & + & y & - & z & = & 3 \\ x & + & 2y & - & 3z & = & 5 \\ 3x & + & 4y & - & 5z & = & 11 \end{array}$$

15.
$$\begin{array}{rcrcrcr} x & + & 3y & - & z & = & 5 \\ 2x & + & y & + & 3z & = & -2 \\ 4x & + & 7y & + & z & = & -1 \end{array}$$

16.
$$\begin{array}{rcrcrcr} 2x & + & 3y & - & z & = & 6 \\ x & - & y & + & 3z & = & 8 \\ 4x & + & y & + & 5z & = & 22 \end{array}$$

17. In WinPlot open the 3-dim window and solve each of the following three equations for $z$. Enter each formula for $z$ as an Explicit function in Equa/Explicit menu. From View pick Axes to get a better 3D perspective of the surfaces. Be sure to use a different color for each surface. This is chosen under color of the Equa/Explicit menu. Solve this system of equations and enter the coordinates of the solution using the Equ/Point/Cartesian menu. Be sure the color of this point is distinct from the color of the three surfaces. Use PgUp (or the View Memu) to "zoom in" and the left and right arrow keys to rotate the graph until you get a "good view" which clearly shows the three surfaces and the solution point. Print and attach your graph.

$$\begin{array}{rcrcrcr} x & - & y & - & z & = & 1 \\ x & + & 2y & + & z & = & -1 \\ 2x & - & y & + & 2z & = & 8 \end{array}$$

What kind of geometric surfaces does each equation describe?

What is the significance of the solution point with respect to the three surfaces?

18. In WinPlot open the 3-dim window and solve each of the following three equations for $z$. Enter each formula for $z$ as an Explicit function in Equa/Explicit menu. From View pick Axes to get a better 3D perspective of the surfaces. Be sure to use a different color for each surface. Show that this system is consistent but dependent with infinitely many solutions along a line. Determine the form of this solution. In this form let $z = t$ where $t$ is any real number. Solve for $x$ and $y$ in terms of $t$. In WinPlot use the Equa/Curve menu to enter the form of the infinetly many solutions. Set t lo = $-5$ and t hi = 5. Set the pen width to 2 and choose a dominant color to make this line more prominent. Use Use PgUp (or the View Memu) to "zoom in" and the left and right arrow keys rotate the graph until you get a "good view" which clearly shows the three surfaces and the line of solutions. Print and attach your graph.

$$\begin{array}{rcrcrcr} x & - & y & - & z & = & 1 \\ x & + & 2y & + & z & = & -1 \\ 3x & + & 3y & + & z & = & -1 \end{array}$$

What kind of geometric surfaces does each equation describe?

What is the significance of the line of solutions with respect to the three surfaces?

Evaluate the following determinants.

19. $\begin{vmatrix} 3 & -5 \\ 4 & 7 \end{vmatrix}$ = _____

20. $\begin{vmatrix} 2 & 1 & -3 \\ 1 & -2 & -1 \\ 3 & 2 & -1 \end{vmatrix}$ = _____

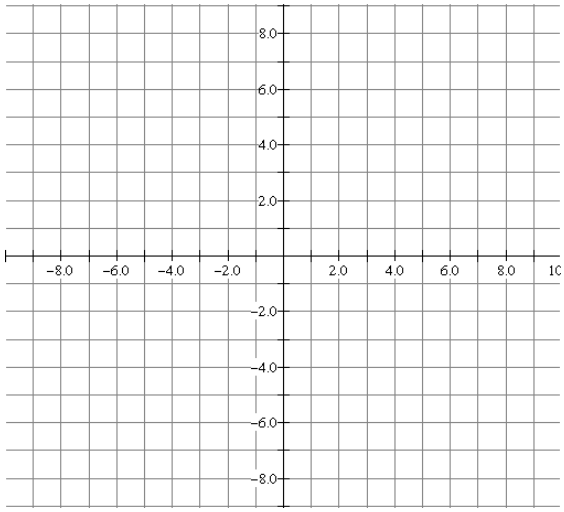
21. $\begin{vmatrix} a & 0 & 0 \\ b & b & -a \\ a & a & b \end{vmatrix}$ = _____

Solve the following systems of equations by Cramer's Rule.

22.
$$\begin{array}{rcrcr} 3x & + & 2y & = & 4 \\ 5x & - & y & = & 11 \end{array}$$

23.
$$\begin{array}{rcrcrcr} x & + & y & + & 2z & = & 1 \\ 2x & - & 2y & - & z & = & -7 \\ x & & & + & 2z & = & 0 \end{array}$$

24. Graph the solution set of the system of inequalities :
$$\begin{array}{rcrcr} -x & + & 2y & \leq & 4 \\ 2x & + & y & > & 7 \end{array}$$



25. Graph the solution set of the system of inequalities :

$$\begin{array}{rcrcr} x & - & y & > & 2 \\ 2x & + & y & < & 11 \\ & & x & > & 0 \\ & & y & > & 0 \end{array}$$