Name ________________________ /100

Each Problem is worth 25 points.

1. The probability that a newly purchased cell phone proves defective within the first six months after purchase is 5%. On January 1, 2009 a local wireless outlet sold 15 such phones.

a) What is the probability that by July 1, 2009 two or more such phones are defective?

Nation wide, 4,457 were sold on January 1, 2009.

b) What is the probability that by July 1, 2009, 240 or more such phones are defective?

c) What is the probability that by July 1, 2009, 200 or less such phones are defective?

d) What is the probability that by July 1, 2009 the number of such phones that prove defective is greater than or equal to 210 but less than or equal to 230?

e) The probability that by July 1, 2009 more than *W* cell phones fail is 75%. Determine the value of *W*.

2. The following distribution (cumulative probability distribution) function $F(x)$ gives the probability that the value of the random variable is less than or equal to *x*.

For $a > 0$, $$F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ \left(\frac{x}{a}\right)^2 & \text{if } 0 < x < a \\ 1 & \text{if } x \ge a \end{cases}$$

a) Determine the probability density function $f(x)$

b) Determine the expected or expectation value of *x*: $E(x) = \langle x \rangle$

c) Determine the expected or expectation value of x^2: $E(x^2) = \langle x^2 \rangle$

d) Determine μ_x

e) Determine σ_x

$f(x) =$ ______________________________

$\langle x \rangle =$ ____________________ $\mu_x =$ ____________________

$\langle x^2 \rangle =$ ____________________ $\sigma_x =$ ____________________

f) Compute the probability that the random variable takes on a value between $\frac{a}{4}$ and $\frac{3a}{4}$.

3. The joint probability density function for two random variables *x* and *y* is given by

$$f(x,y) = \begin{cases} \frac{1}{\alpha\beta} & \text{if } 0 \le x \le \alpha \text{ and } 0 \le y \le \beta \\ 0 & \text{elsewhere} \end{cases}.$$

a) Determine the probability density function of *x*, $f_x(x)$.

b) Determine the mean and standard deviation of *x*.

$f_x(x) =$ ______________________________

$\mu_x =$ ____________________

$\sigma_x =$ ____________________

c) Determine the probability density function of *y*, $f_y(y)$.

d) Determine the mean and standard deviation of *y*.

$f_y(y) =$ ______________________________

$\mu_y =$ ____________________

$\sigma_y =$ ____________________

e) Explain whether or not *x* and *y* are independent random variables.

f) Compute the probability of the event that $\frac{\alpha}{2} \le x \le \alpha$ and $0 \le y \le \frac{2\beta}{3}$.

g) Compute the probability of the event that $\frac{\alpha}{2} \le x \le \alpha$ or $0 \le y \le \frac{2\beta}{3}$

h) Compute the Expectation value of $4x - 3y + 2\alpha$.
$\langle 4x - 3y + 2\alpha \rangle =$ __________________________________

i) Compute the population variance of $4x - 3y + 2\alpha$.
$\sigma^2_{4x-3y+2\alpha} =$ __________________________________

4. A manufactured part has a mean mass of 120. g and a standard deviation of 1.5 g .
Nine components are randomly sampled with replacement.

a) What is the expected value of the sum of masses of these nine components?

b) What is the variance of the sum of the masses of these nine components?

c) What is the expected value of the sample mean of the masses for these nine components?

d) What is the variance of the sample mean of masses of these nine components?